PARISHRAM PUBLICATIONS

Std.: X (English)	Mathematics Part - II	Marks: 40
Date: 14-Dec-2019	Parishram Academy	Time: 2 hrs
	Chapter: All	

Note:-

Q.1 A) Solve Multiple choice questions. **(4)**

1) $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$ is equal to

a. -2 b. 0 c. 1 d. 2

Ans. Option d.

2) A circle touches all sides of a parallelogram. So the parallelogram must be a ....

a. rectangle b. rhombus c. square d. trapezium

Ans. Option b.

3) Find the ratio of the volumes of a cylinder and a cone having equal radius and equal height.

a. 1 : 2 b. 2 : 1 c. 1 : 3 d. 3 : 1

Ans. Option d.

4) If in two triangles ABC and PQR,

$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

a. $\triangle PQR \sim \triangle CAB$ B. $\triangle PQR \sim \triangle ABC$ C. $\triangle CBA \sim \triangle PQR$ D. $BCA \sim \triangle PQR$

Ans. Option a.

B) Solve the following questions. **(4)**

1) Identify, with reason, if the following is Pythagorean triplet. 4, 9, 12

Ans. (4, 9, 12)

$12^2 = 144$

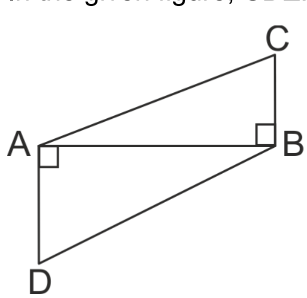
and $4^2 + 9^2 = 16 + 81$

$= 97$

$\therefore \quad 12^2 \neq 4^2 + 9^2$

$\therefore$ (4, 9, 12) is not a Pythagorean triplet.

2) In the given figure, $CB \perp AB$, $DA \perp AB$. If BC = 4, AD = 8 then $\frac{A(\triangle ABC)}{A(\triangle ADB)}$ find.



Ans. $\frac{A(\triangle ABC)}{A(\triangle ADB)} = \frac{BC}{AD}$... (Triangles with same base)

$$\frac{A(\triangle ABC)}{A(\triangle ADB)} = \frac{4}{8}$$
$$\frac{A(\triangle ABC)}{A(\triangle ADB)} = \frac{1}{2}$$

3) Area of a sector of a circle of radius 15 cm is 30 cm^2. Find the length of the arc of the sector.

Ans. Given : Area of sector = 30 cm^2

Radius of circle = 15 cm

To find : length of arc

Solution :

area of sector $= \frac{\text{Length of arc} \times \text{radius of circles}}{2}$

$\therefore \quad 30 = \frac{\text{length of the arc} \times 15}{2}$

$\frac{30 \times 2}{15}$

$= 4$ cm

Length of the arc is 4 cm

4) Prove the following

$\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Ans. $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

LHS = $\tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$

$= \tan^2\theta \cdot \sec^2\theta$ ... $[1 + \tan^2\theta = \sec^2\theta]$

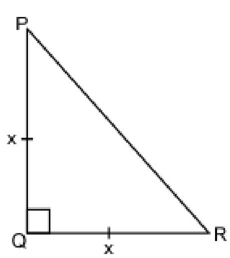
$= (\sec^2\theta - 1)\sec^2\theta$

$= \sec^4\theta - \sec^2\theta$

$\therefore$ LHS = RHS

Q.2 A) Complete the following Activities. (Any two) **(4)**

1) A side of an isosceles right angled triangle is x. Find its hypotenuse.



In $\triangle PQR$, $\angle PQR = 90°$

and PQ = QR = x

$\therefore$ PR^2 = ______ ... [Pythagoras theorem]

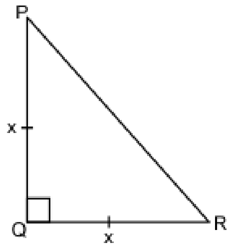
= ______

$\therefore$ PR^2 = ______

$\therefore$ PR = ______units ... [Taking square root]

$\therefore$ The length of hypotenuse is ______ units.

Ans. A side of an isosceles right angled triangle is x. Find its hypotenuse.



In $\triangle PQR$, $\angle PQR = 90^\circ$

and $PQ = QR = x$

$\therefore$ $PR^2 = PQ^2 + QR^2$ ... [Pythagoras theorem]

$= x^2 + x^2$

$\therefore$ $PR^2 = 2x^2$

$\therefore$ $PR = \sqrt{2}x$ units ... [Taking square root]

$\therefore$ The length of hypotenuse is $\sqrt{2}$ x units.

2) If $\sin\theta = \frac{11}{61}$ then find the value of $\cos\theta$ using identity.

$\sin^2\theta + \cos^2\theta = 1$ ... [Trigonometric identity]

$\therefore$ $\cos^2\theta =$ ______

$= 1 -$ ______

$= 1 - \frac{121}{3721}$

$=$ ______

$\therefore$ $\cos^2\theta =$ ______

$\therefore$ $\cos\theta =$ ______ ... [Taking square root]

Ans. If $\sin\theta = \frac{11}{61}$ then find the value of $\cos\theta$ using identity.

$\sin^2\theta + \cos^2\theta = 1$ ... [Trigonometric identity]

$\therefore$ $\cos^2\theta = 1 - \sin^2\theta$

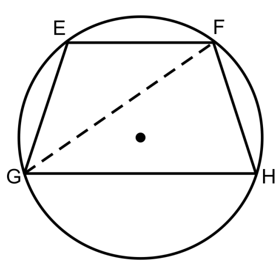
$= 1 - \left(\frac{11}{61}\right)^2$

$= 1 - \frac{121}{3721}$

$= \frac{3721 - 121}{3721}$

$\therefore$ $\cos^2\theta = \frac{3600}{3721}$

$\therefore$ $\cos\theta = \frac{60}{61}$ ... [Taking square root]

3) 

In chord EF || chord GH. Prove that, chord EG $\cong$ chord FH.
Fill in the blanks and write the proof.

Proof : Draw seg GF.

$\angle EFG = \angle FGH$ ... .(I)

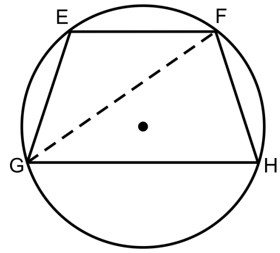
$\angle EFG =$ .... inscribed angle theorem (II)

$\angle FGH =$ .... inscribed angle theorem (III)

∴ m(arc EG) = ... from (I), (II), (III).

∴ chord EG ≅ chord FH ... .

Ans.



In chord EF || chord GH. Prove that, chord EG ≅ chord FH.

Fill in the blanks and write the proof.

Proof : Draw seg GF.

∠EFG = ∠FGH ... [alternate angles] (I)

∠EFG = $\frac{1}{2}\times m\left(\text{arc EG}\right)$ ... inscribed angle theorem (II)

∠FGH = $\frac{1}{2}\times m\left(\text{arc FH}\right)$ ... inscribed angle theorem (III)

∴ m(arc EG) = m(arc FH) ... from (I), (II), (III).

∴ chord EG ≅ chord FH ... [congruent arcs of congruent chords]

B) Solve the following questions. (Any four) **(8)**

1) The diameter of a circle is 10 cm. Find the length of the arc, when the corresponding central angle is 144° (π = 3.14).

Ans. Given : Diameter = 10 cm

radius = $\frac{\text{diameter}}{2}=\frac{10}{2}=5$

∴ radius (r) = 5 cm

Central angle (θ) = 144

length of arc = $\frac{\theta}{360}\times 2\pi r$

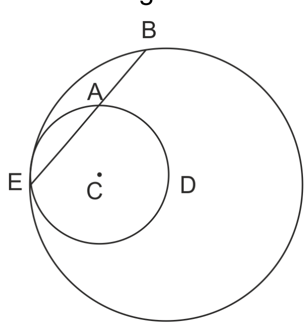
length of arc = $\frac{144}{360}\times 2\times 3.14\times 5$

length of arc = $\frac{2}{5}\times 2\times 3.14\times 5$

length of arc = 12.56

∴ length of arc is 12.56

2)



In the figure circles with centres C and D touch internally at point E. D lies on the inner circle. Chord EB of the outer circle intersects inner circle at point A. Prove that, seg EA ≅ seg AB.

Ans. Proof :

Draw seg ED and seg DA.

E–C–D ... (By theorem of touching circles)

∴ seg ED is the diameter of the smaller circle.

$\angle EAD = 90^\circ$ ... (Angle inscribed in a semicircle is a right angle) ... (1)

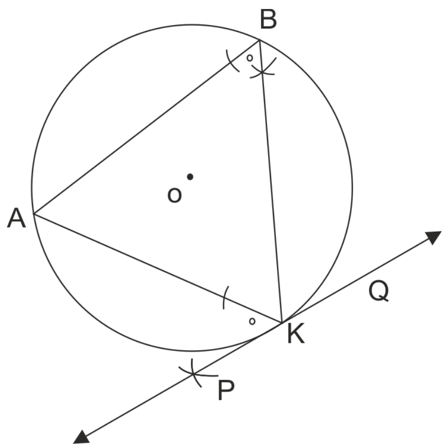
For larger circle, seg DA $\perp$ chord EB ... [From (1)]

$\therefore$ EA = AB ... (Perpendicular drawn from the centre of the circle to the chord bisects the chord)

$\therefore$ **seg EA $\cong$ seg AB**

3) Draw a circle of radius 3.6. Draw a tangent to the circle at any point on it without using centre.

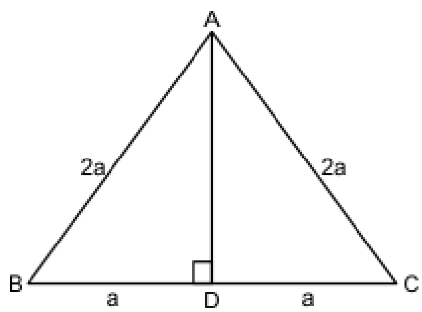
Ans.



Line BN is the required tangent.

4) Find the length of altitude of an equilateral triangle having side 2a.

Ans.



$\triangle ABC$ is an equilateral triangle having each side 2a units.

seg AD $\perp$ side BC.

$\therefore$ $BD = \frac{1}{2} BC$ ... [In an equilateral triangle, altitude is also median]

$= \frac{1}{2} \times 2a$

$\therefore$ BD = a units

In $\triangle ADB$, $\angle ADB = 90°$

$\therefore$ $AB^2 = AD^2 + BD^2$ ... [Pythagoras theorem]

$\therefore$ $(2a)^2 = AD^2 + (a)^2$

$\therefore$ $4a^2 - a^2 = AD^2$

$\therefore$ $AD^2 = 3a^2$

$\therefore$ AD = $\sqrt{3}$ a units

$\therefore$ Length of altitude is $\sqrt{3}$a units.

5) Find the centroids of the triangles whose vertices are given below.
(3, − 5), (4, 3), (11, − 4)

Ans. Let A (3, – 5) ≡ (x_1, y_1),
B (4, 3) ≡ (x_2, y_2) and
C (11, – 4) ≡ (x_3, y_3)

Let G ≡ (x, y) be the centroid of △ABC.

By centroid formula,

$x = \frac{x_1 + x_2 + x_3}{3}$, $y = \frac{y_1 + y_2 + y_3}{3}$

$= \frac{3 + 4 + 11}{3}$ $= \frac{-5 + 3 - 4}{3}$

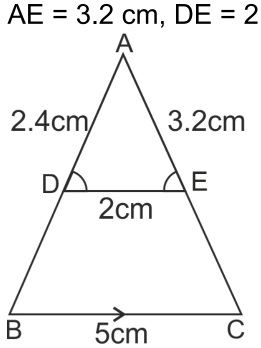
$= \frac{18}{3}$ $= \frac{-6}{3}$

$= 6$ $= -2$

$\therefore$ The coordinates of the centroid of △ ABC are (6, – 2).

Q.3 A) Complete the following activity. (Any one) **(3)**

1) In a △ABC, D and E are points on the sides AB and AC respectively such that DE ∥ BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find the BD and CE.



In △ABC

$\because$ DE ∥ BC

$\therefore$ $\frac{AD}{AB}$ = ______

$\frac{2.4}{AB} = \frac{3.2}{AC} = \frac{2}{5}$

$\frac{2.4}{AB} = \frac{2}{5}$

AB = ______

= 6cm

$\frac{3.2}{AC} = \frac{2}{5}$

and AC = ______ = 8cm

$\therefore$ BD = ______

= 6 - 2.4

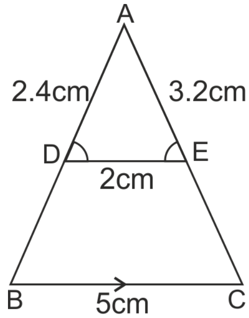
= ______

and CE = AC - AE

= 8 - 3.2

= ______

Ans. In a $\triangle ABC$, D and E are points on the sides AB and AC respectively such that DE $\parallel$ BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find the BD and CE.



In $\triangle ABC$

$\because$ DE $\parallel$ BC

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

$\frac{2.4}{AB} = \frac{3.2}{AC} = \frac{2}{5}$

$\frac{2.4}{AB} = \frac{2}{5}$

AB = $\frac{2.4 \times 5}{2}$

= 6cm

$\frac{3.2}{AC} = \frac{2}{5}$

and AC = $\frac{3.2 \times 5}{2}$ = 8cm

$\therefore$ BD = AB - AD

= 6 - 2.4

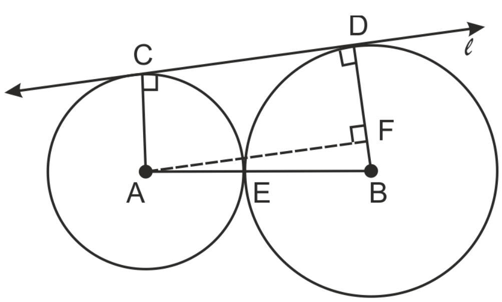
= 3.6cm

and CE = AC - AE

= 8 - 3.2

= 4. 8cm

2) In the circles with centres A and B touch each other at E. Line l is a common tangent which touches the circles at C and D respectively. Find the length of seg CD if the radii of the circles are 4 cm, 6 cm.



Construction: Draw seg AF $\perp$ seg BD

$\therefore$ $\square$AFDC is a rectangle.

A - E - B ... [_____]

$\therefore$ _____ = AB ... [A - E - B]

4 + 6 = AB

AB = 10cm

Now, in ΔAFB, $\angle AFB$ = 90º .... [Construction]

AB^2 = _____ .... [Pythagoras Theorem]

$\therefore$ $10^2 = AF^2 + 2^2$ BF = _____

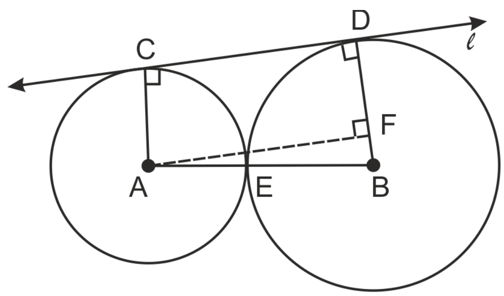
$AF^2 = 96$

∴ AF = ______

But, CD = AF

∴ CD = ______

Ans. In the circles with centres A and B touch each other at E. Line l is a common tangent which touches the circles at C and D respectively. Find the length of seg CD if the radii of the circles are 4 cm, 6 cm.



Construction: Draw seg AF $\perp$ seg BD

∴ $\square AFDC$ is a rectangle.

A - E - B ... [If two circles are touching circles then their point of contact lies on the line joining their centres]

∴ AE + EB = AB ... [A - E - B]

4 + 6 = AB

$\boxed{\mathbf{AB = 10cm}}$

Now,

in ΔAFB, $\angle AFB = 90°$ .... [Construction]

$AB^2 = AF^2 + BF^2$ .... [Pythagoras Theorem]

∴ $10^2 = AF^2 + 2^2$ BF = BD - DF

$AF^2 = 96$

∴ AF = $4\sqrt{6}$

But, CD = AF

∴ CD = $4\sqrt{6}$

B) Solve the following questions. (Any two) **(6)**

1) Prove the following.

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A}$$

Ans. $$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{1 - 2\cos^2 A} = \frac{2\sec^2 A}{\tan^2 A - 1}$$

L.H.S. = $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

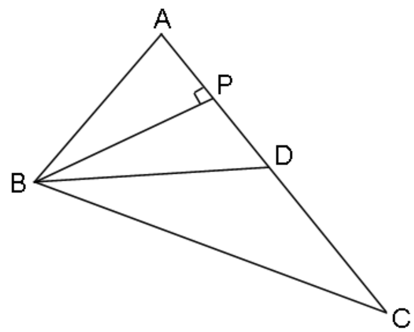
= $\frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\sin^2 A - \cos^2 A}$

= $\frac{2(\sin^2 A + \cos^2 A)}{\sin^2 A - \cos^2 A}$

= $\frac{2 \times 1}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - \cos^2 A}$

= R.H.S.

2)



In adjoining figure in $\triangle ABC$, point D is on side AC. If AC = 16, DC = 9 and BP $\perp$ AC, then then find the following ratios.

i. $\frac{A(\triangle ABD)}{A(\triangle ABC)}$ ii. $\frac{A(\triangle BDC)}{A(\triangle ABC)}$ iii. $\frac{A(\triangle ABD)}{A(\triangle BDC)}$

Ans. In $\triangle ABC$ point P and D are on side AC,
hence B is a common vertex of $\triangle ABD$, $\triangle BDC$, $\triangle ABC$ and $\triangle APB$ and their sides AD, DC, AC and AP are collinear.
Heights of all the triangles are equal.
Hence, areas of these triangles are proportional to their bases AC = 16, DC = 9

$\therefore$ AD = 16 - 9 = 7

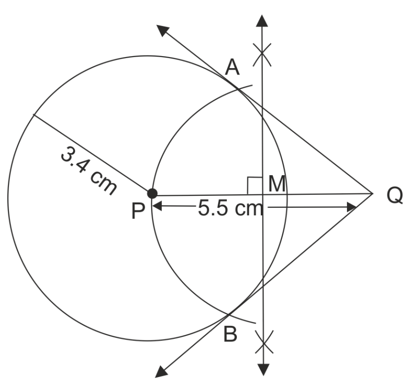
$\therefore$ $\frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{AD}{AC} = \frac{7}{16}$ ... triangles having equal heights

$\frac{A(\triangle BDC)}{A(\triangle ABC)} = \frac{DC}{AC} = \frac{9}{16}$ ... triangles having equal heights

$\frac{A(\triangle ABD)}{A(\triangle BDC)} = \frac{AD}{DC} = \frac{7}{9}$ ... triangles having equal heights

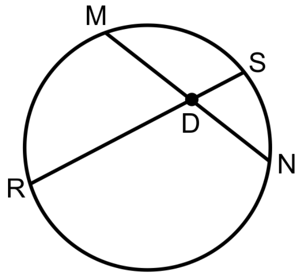
3) Draw a circle with centre P and radius 3.4 cm. Take point Q at a distance 5.5 cm from the centre. Construct tangents to the circle from point Q.

Ans.



Line QA and line QB are required tangents.

4) In figure, chord MN and chord RS intersect at point D.
(1) If RD = 15, DS = 4, MD = 8 find DN
(2) If RS = 18, MD = 9, DN = 8 find DS



Ans. Case (i) :

Given : RD = 15, DS = 4

MD = 8

To find : DN = ?

Solution :

$MD \times DS = RD \times DS$ ... {Property on intersecting chords}

$8 \times DN = 4 \times 15$

$\therefore$ $DN = \frac{15}{2}$

DN = 7.5 units

Case (ii) :

Given : RS = 18, MD = 9, DN = 8

To find : DS

Solution :

Let DS = x

RD + DS = RS ... {R-D-S}

$RD = RS - DS$

$RD = 18 - x$

Now,

$MD \times DN = RD \times DS$ ... {By property of intersecting chords}

$8 \times DN = (18 \times x) \times x$

$8 \times 9 = (18 - x) \times x$

$\therefore$ $18x - x^2 = 72$

$x^2 - 18x + 72 = 0$

$x^2 - 12x - 6x + 72 = 0$

$x(x - 12) - 6(x - 12)$

$(x - 6)(x - 12)$

$\therefore$ x = 6 Or x = 12

$\therefore$ **DS = 6 units** Or

DS = 12 units

Q.4 Solve the following questions. (Any two) (8)

1) Find the equation of the line passing through the point of intersection of the line 4x + 3y + 2 = 0 and 6x + 5y + 6 = 0 and the point of intersection of the lines 4x - 3y - 17 = 0 and 2x + 3y + 5 = 0.

Ans. To find the points of intersections of the given pairs of lines, we have to solve the equations.

$4x + 3y + 2 = 0$ $\therefore$ $4x + 3y = -2$ ... (1)

$6x + 5y + 6 = 0$ $\therefore$ $6x + 5y = -6$ ... (2)

Multiplying equation (1) by 5 and equation (2) by 3.

-18x+15y=-182x=8 ... (3)... (4) ... [Subtracting eq. (4) from eq. (3)]

$\therefore$ x = 4

Substituting $x = 4$ in equation (1),

$4(4) + 3y = - 2$

∴ $16 + 3y = - 2$ ∴ $3y = - 2 - 16$

∴ $3y = - 18$ ∴ $y = - 6$

∴ the coordinates of the point of intersection of the first pair of lines are (4, - 6)

Let P $(4, - 6) \equiv (x_1, y_1)$ ... (a)

For the second pair of lines,

$4x - 3y - 17 = 0$ ∴ $4x - 3y = 17$ ... (i)

$2x + 3y + 5 = 0$ ∴ $2x + 3y = - 5$ ... (ii)

Adding equations (i) and (ii),

$6x = 12$ ∴ $x = 2$

Substituting $x = 2$ in equation (ii),

$2(2) + 3y = - 5$

∴ $4 + 3y = - 5$ ∴ $3y = - 5 - 4$

∴ $3y = - 9$ ∴ $y = - 3$

∴ the coordinates of the point of intersection of the second pair of lines are (2, - 3).

Let Q $(2, - 3) \equiv (x_2, y_2)$ ... (b)

The equation of a line in two-point form is

x - x1x1 - x2 = y - y1y1 - y2

∴ the equation of line PQ is

x - 44 - 2 = y - (- 6)- 6 - (- 3) ... [From (a) and (b)]

∴ x - 42 = y -(-6)- 6 + 3 ∴ x - 42 = y + 6- 3

∴ $- 3 (x - 4) = 2 (y + 6)$

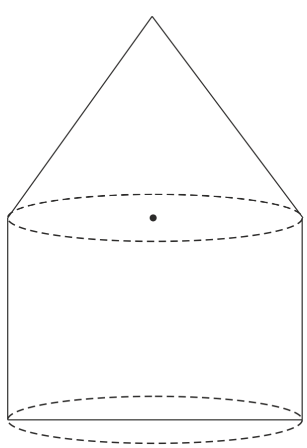
∴ $- 3x + 12 = 2y + 12$

∴ $- 3x - 2y = 12 - 12$ ∴ $- 3x - 2y = 0$

i.e. $3x + 2y = 0$

The equation of the required line is $3x + 2y = 0$

2)



A cylinder and a cone have equal bases. The height of the cylinder is 3 cm and the area of its base is 100 cm^2.The cone is placed upon the cylinder. Volume of the solid figure so formed is 500 cm^3. Find the total height of the figure.

Ans. Given : radius of cylinder = radius of cone = r

height of cylinder = h_{cy} = 3 cm

Area of base = 100 cm^2

Volume of solid = 500 cm^3

To find : Total height of figure

Solution : Volume of cylinder $= \pi r2h$

$= \text{Area of base} \times \text{height of cylinder}$

$= 100 \times 3$

Volume of cylinder $= 300\ cm^3$ ...(1)

Total volume = Volume of cone + Volume of cylinder

$\therefore$ Volume of cone = Volume of solid - Volume of cylinder

$= 500 - 300$ ... [given and (1)]

Volume of cone $= 200\ cm^3$

By formula of volume of cone

$13 \times \pi \times r2 \times h = 200$

$\therefore$ $13 \times \text{Area of base} \times h = 200$

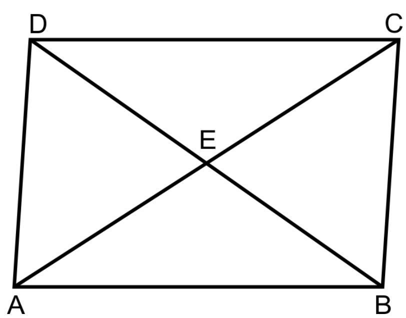
$\therefore$ $13 \times 100 \times h = 200$ ... (given)

$\therefore$ Total height = 9 cm

Total height of solid is 9 cm.

3) Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Ans.



□ABCD is a prallelogram. Diagonals AC and BD intersect each other at point E.

To prove:

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

proof:

i. □ABCD is a parallelogram ... [given]

$\therefore$ AE = EC and BE = DE ... [Diagonals of parallelogram bisect each other]

ii. AB = CD and BC = AD ... [Opposite sides of parallelogram are equal]

iii. In △ABC, seg BE is the median ... [From (1)]

$\therefore$ $AB^2 + BC^2 = 2\ BE^2 + 2\ AE^2$ ... [Apollonius theorem]

iv. $\therefore$ $AB^2 + BC^2 = 2\ 12\ BD2 + 2\ 12\ AC2$ ... [From (1), (3)]

$= 2 \times 14\ BD^2 + 2 \times 14\ AC^2$

$= 12\ BD^2 + 12\ AC^2$

$\therefore$ $AB^2 + BC^2 = 12(BD^2 + AC^2)$

$\therefore$ $2\ (AB^2 + BC^2) = BD^2 + AC^2$ ... [Multiplying both side by

$\therefore$ $2\ AB^2 + 2\ BC^2 = BD^2 + AC^2$

v. $AB^2 + AB^2 + BC^2 + BC^2 = BD^2 + AC^2$ ... [From (4)]

$\therefore$ $AB^2 + CD^2 + BC^2 + AD^2 = BD^2 + AC^2$ ... [From (2)]

$\therefore$ $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

Q.5 Solve the following questions. (Any one) (3)

1) A building has 8 right cylindrical pillars whose cross sectional diameter is 1 m and whose height is 4.2 m. Find the expenditure to paint these pillars at the rate of Rs. 24 per m^2.

Ans. **Given:** For the cylindrical pillars:

diameter = 1m

radius (r) = diameter2 = 12m

height (h) = 4.2 m

rate of painting = Rs. 24/m^2

To find: Cost of painting the pillars

Curved surface area of each cylindrical pillar

= 2πrh

= 2×227×12×4.2

= 13.2m^2

Area of one pillar to be painted = 13.2 m^2

Total area to be painted

= Number of pillars × curved surface area each pillar

= 8 × 13.2

Total area to be painted = 105.6 m^2

Rate of painting = Rs 24/ m^2

Cost of painting

= Total area to be painted ×rate of painting

= 105.6 × 24 = Rs 2534.40

∴ **The expenditure to paint the pillars Rs. 2534.40.**

2) Find the coordinates of point P if P divides the line segment joining the points.
A (-1,7) and B (4,- 3) in the ratio 2 : 3.

Ans. Let A (- 1, 7)≡(x_1, y_1),

B (4, - 3)≡(x_2, y_2) and

P≡(x_1, y)

Herex_1 = – 1, y_1 = 7, x_2 = 4,

y_2 = - 3, m = 2 and n = 3

By section formula

x = mx2 + nx1m + n , y = my2 + ny1m + n

= 2 (4)+3 (-1)2 + 3 = 2(-3) + 3(7)2+3

= 8-35 = - 6+215

= 55 = 155

∴ x = 1 ∴ y = 3

∴ The coordinates of point P are (1, 3).