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FINAL EXAMINATION
YEAR 2019-20
Std. : X (CBSE)
Subject : Mathematics
MODEL ANSWER

X-All/
Dt.26.12.2019

A

Section - A

1) (a) Since, -3 is the zero of the given quadratic polynomial

∴ $(k - 1)(-3)^2 + k(-3) + 1 = 0$ ⇒ $(k - 1) \times 9 - 3k + 1 = 0$

⇒ $9k - 9 - 3k + 1 = 0$ ⇒ $6k - 8 = 0$

∴ $k = \frac{4}{3}$ (1)

2) (c) Given $\sqrt{3}\tan\theta = 1$ ⇒ $\tan\theta = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow \theta = 30°$ ($^1/_2$)

∴ $\sin^2\theta - \cos^2\theta = \sin^2 30^o - \cos^2 30^o$ $= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$ ($^1/_2$)

3) (d) We know that composite number are those number which has atleast one factor other 1 and the number itself. Number 3, 5 and 7 has no other factor, so it is not composite number, number 9 is composite number, because it has factor 3 x 3. (1)

4) (a) In a dice, the number greater than 4 is (5,6).

∴ The number of favourable outcomes = 2

Total number of outcomes = 6

∴ Probability of getting a number greater than

$4 = \frac{2}{6} = \frac{1}{3}$ (1)

5) (c) We know that, Mode = 3 Median - 2 Mean. ⇒ $\text{Median} = \frac{\text{Mode} + 2\,\text{Mean}}{3}$ ($^1/_2$)

∴ $\text{Median} = \frac{45 + 2 \times 27}{3} = \frac{99}{3} = 33$ ($^1/_2$)

6) (a) Let the required ratio k : 1. Then, $\frac{7k-2}{k+1} = 1$ ⇒ $7k - 2 = k + 1 \Rightarrow 6k = 3$

∴ $k = \frac{1}{2} = 1:2$ (1)

7) (c) We have, r = 21 cm, = 60 ⇒ ∵ $l = \frac{\theta}{360°} \times 2\pi r$ ($^1/_2$)

∴ $l = \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21 = 22\,\text{cm}$ ($^1/_2$)

8) (a) $x + 1 - (x - 1) = (2x + 3) - (x + 1)$ ⇒ $2 = x + 2 \Rightarrow x = 0$ (1)

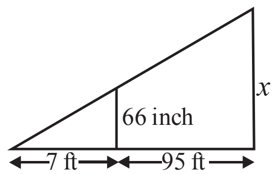
9) (a) Given, $\triangle ABC \sim \triangle DEF$

$\Rightarrow \quad \dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \dfrac{AB}{DE}$ [∵ the ratio of the perimeter of two similar triangles is the same as the ratio of their corresponding sides] ($^1/_2$)

Let p be the perimeter of $\triangle ABC$.

Then, $\dfrac{p}{25} = \dfrac{9.1}{6.5} = \dfrac{7}{5}$ ∴ $p = 35$ cm ($^1/_2$)

10) (a) ($^1/_2$)



Height of Rubal $= \dfrac{66}{12}\text{ ft} = 5.5\text{ ft}$ [∵ 1 ft = 12 inches]

So, we get $\dfrac{7}{7+95} = \dfrac{5.5}{x}$ [By similarity theorem]

$\Rightarrow \quad \dfrac{7}{102} = \dfrac{5.5}{x} \Rightarrow 7x = 102 \times 5.5 \Rightarrow x = \dfrac{561}{7}$ ∴ $x = 80$ ft [approx] ($^1/_2$)

11) (-2,1) Here, $(x_1, y_1) = (-8, 0)$, $(x_2, y_2) = (5, 5)$ and $(x_3, y_3) = (-3, -2)$

∴ Centroid of $\triangle PQR$

$= (x, y) = \left[\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right]$ ($^1/_2$)

$= \left[\dfrac{-8+5-3}{3}, \dfrac{0+5-2}{3}\right] = (-2, 1)$ ($^1/_2$)

12) (Euclid's Division Lemma). Given statement is known as Euclid's Division Lemma (1)

13) (Parallel) Two tangents, drawn at the end points of diameter of a given circle are always parallel. (1)

14) ($\angle Q$). If $\triangle ABC \sim \triangle PQR$, then their corresponding angles are equal. Therefore $\angle B = \angle Q$. (1)

15) (sin θ) cos (90° - θ) = sin θ. (1)

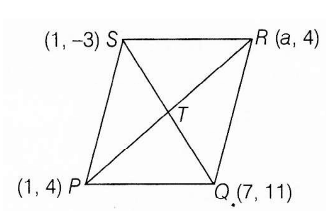
16) Let x be the HCF of the numbers. We have the numbers as $3x$, $4x$ and $5x$

Now, LCM $(3x, 4x, 5x) = 3 \times 4 \times 5 \times x = 60x$

$\Rightarrow \quad 60x = 1200$ ∴ $x = 20$ (1)

17) It is clear that the graph of $p(x)$ cut the x-axis at only one point. Hence, the number of zeroes of $p(x)$ is 1. (1)

18) Let $P(1, 4)$, $Q(7, 11)$, $R(a, 4)$ and $S(1, -3)$ be the vertices of a parallelogram $PQRS$ respectively. Join PR and QS. Let PR and QS intersect at the point T.



($^1/_2$)

We know that, the diagonals of a parallelogram bisect each other. So, T is the mid-point of PR as well as that of QS.

∴ Mid-point of PR = Mid-point of QS

$\Rightarrow \quad \left(\frac{1+a}{2},\frac{4+4}{2}\right)=\left(\frac{7+1}{2},\frac{11-3}{2}\right) \quad \Rightarrow \quad \left(\frac{1+a}{2},4\right)=(4,4)$

On comparing x - coordinate from both sides, we get

$\frac{1+a}{2}=4 \Rightarrow 1+a=8 \Rightarrow a=7$ Hence, the value of a is 7. (1/2)

19) Given, AP is 21,18,15,...

Here a =21 and d = 18 - 21 = -3. Let nth term of given AP be -81.

Then, $a_n = -81$

$\Rightarrow a + (n - 1)d = -81$ (1/2)

$[a_n = a+(n-1)d]$ ----- (i)

On putting the values of a and d in eq. (i) we get

$21 + (n - 1) \times (-3) = -81$

$\Rightarrow 21 - 3n + 3 = -81$

$\Rightarrow 24 - 3n = -81 \Rightarrow 3n = -81 - 24$

$\therefore n=\frac{-105}{-3}=35$ (1/2)

Hence, 35th terms of given AP is -81.

20) Given system of equations is

$x + ky = 0$ and $2x - y = 0$

On comparing these equations with

$a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0,$ (1/2)

we get $a_1 = 1, b_1 = k, c_1 = 0$

and $a_2 = 2, b_2 = -1, c_2 = 0$

condition for unique solution

$\frac{a1}{a2}\neq\frac{b1}{b2}\Rightarrow\frac{1}{2}\neq\frac{k}{-1}\Rightarrow k\neq-\frac{1}{2}$ (1/2)

Section - B

21) Given pair of linear equations

$x + y = 14$ ----- (i)

$x - y = 4.$ ----- (ii) (1/2)

From (ii), $x = 4 + y$ ----- (iii)

Substitute this value of x in eq. (i), we get

$4 + y + y = 14$

or $2y = 14 - 4$

or $2y = 10$

or $y=\frac{10}{2}=5$ (1/2)

Substitute this value of y in eq. (iii) we get

$x = 4 + 5 = 9$

Hence, $x = 9$ and $y = 5$ (1)

22) $\frac{\sin^2 63^\circ+\sin^2 27^\circ}{\cos^2 17^\circ+\cos^2 73^\circ}$

$=\frac{[\sin^2(90^\circ-27^\circ)+\sin^2 27^\circ]}{\cos^2 17^\circ+\{\cos(90^\circ-17^\circ)\}^2}$ (1/2)

$=\frac{(\cos 27^\circ)^2+\sin^2 27^\circ}{\cos^2 17^\circ+(\sin 17^\circ)^2}$ $[\because \sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta]$ (1/2)

$$=\frac{\cos^2 27^\circ+\sin^2 27^\circ}{\cos^2 17^\circ+\sin^2 17^\circ}=\frac{1}{1}=1$$ (1)

OR

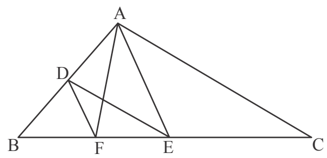
sin 25° cos 65° + cos 25° sin 65°

= sin 25° x cos (90° - 25°) + cos 25° x sin (90° - 25°) ($^1/_2$)

= sin 25° x sin 25° + cos 25° x cos 25° [∵ cos (90° - θ) = sin θ and sin (90° - θ) = cos θ] ($^1/_2$)

= sin² 25° + cos² 25° = 1 (1)

23)



In $\triangle ABC$, $DE \parallel AC$

$\therefore \quad \frac{BD}{DA}=\frac{BE}{EC}$ ----- (i) [By basic proportionality theorem] ($^1/_2$)

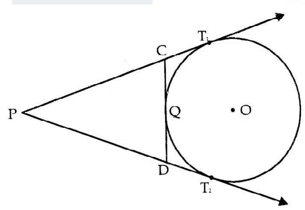
In $\triangle ABE$, $DF \parallel AE$

$\therefore \quad \frac{BD}{DA}=\frac{BF}{FE}$ ----- (ii) [By basic proportionality theorem] ($^1/_2$)

From (i) and (ii)

$$\frac{BE}{EC}=\frac{BF}{FE}$$ (1)

24) ($^1/_2$)



Length of tangents drawn from external point are equal.

Therefore, $PT_1 = PT_2 = 12$ cm & $CQ = CT_1 = 2$ cm ($^1/_2$)

Now, $PC = PT_1 - CT_1 = PC = (12 - 2)$ cm = 10 cm (1)

25) Volume of water in the water tank = 11 x 6 x 5 m³

Let height of the cylindrical tank = h m

∴ Volume of cylindrical tank = $\pi r^2 h=\frac{22}{7}\times(3.5)^2\times h$ (1)

$$\therefore \quad \frac{22}{7}\times(3.5)^2\times h=11\times6\times5$$

$$\Rightarrow \quad h=\frac{11\times6\times5\times7\times10\times10}{22\times35\times35}$$

$$h=\frac{3\times10\times10}{35}=8.6\text{ m}$$ (1)

OR

Volume of solid cube = (44)³ cm³. For spherical lead shots, r = 2 cm

Volume of one lead shot $=\frac{4}{3}\pi(2)^3$ cm³ (1)

Number of lead shots $=\frac{44\times44\times44}{\frac{4}{3}\times\frac{22}{7}\times8}=\frac{22\times44\times21}{8}=2541$ (leadshots) (1)

26) Total number of marbles in jar = 24
Let number of green marbles = x
$\therefore$ Number of blue marbles = 24 - x

Probability of drawing a green marble $= \frac{2}{3}$ $(^1/_2)$

$$\frac{x}{24} = \frac{2}{3}$$ $(^1/_2)$

$$x = \frac{24 \times 2}{3} = 16$$

$\therefore$ Number of green marbles = 16 $(^1/_2)$
$\therefore$ Number of blue marbles = 24 - x = 24 - 16 = 8. $(^1/_2)$

Section - C

27) Given that $p(x) = x^4 - 3x^2 + 4x + 5$
or $p(x) = x^4 + 0x^3 - 3x^2 + 4x + 5$
and $g(x) = x^2 + 1 - x$
or $g(x) = x^2 - x + 1$

$$\begin{array}{r|l} & x^2 + x - 3 \\ \hline x^2 - x + 1 & x^4 + 0x^3 - 3x^2 + 4x + 5 \\ & x^4 - x^3 + 3x^2 \\ & - \quad + \quad - \\ \hline & x^3 - 4x^2 + 4x + 5 \\ & x^3 - x^2 + x \\ & - \quad + \quad - \\ \hline & -3x^2 + 3x + 5 \\ & -3x^2 + 3x - 3 \\ & + \quad - \quad + \\ \hline & 8 \end{array}$$ (2)

By division algorithm $x^4 - 3x^2 + 4x + 5 = (x^2 + x - 3)(x^2 - x + 1) + 8$
Hence, quotient = $x^2 + x - 3$ and remainder = 8. (1)

28) The angles of a triangle are x, y and 40^o.
Therefore, $x + y + 40^o = 180^o$ by angle sum property of a triangle.
So, we obtain corresponding equations as $x + y = 140$ ----- (i) $(^1/_2)$
Also, the difference between the two angles x and y is 30^o.
Therefore, we get the following equations : $x - y = 30$ ----- (ii) $(^1/_2)$
Adding (i) and (ii), we get
$2x = 170$

$\Rightarrow$ $x = \frac{170}{2} = 85$ (1)

Putting the value of x in (i), we get
$\Rightarrow$ $y = 55$ $\therefore$ $x = 85^o$ and $y = 55^o$. (1)

OR

Let numerator of fraction = x, Denominator of fraction = y

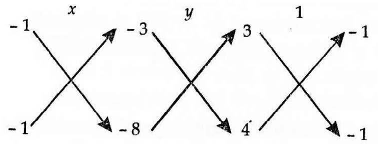
$\therefore$ Required fraction $= \frac{x}{y}$

According to 1st condition,

$\frac{x-1}{y} = \frac{1}{3}$ or $3x - 3 = y$ or $3x - y - 3 = 0$ ---- (i) $(^1/_2)$

According to 2nd condition,

$\frac{x}{y+8} = \frac{1}{4}$ or $4x = y + 8$ or $4x - y - 8 = 0$ ---- (ii) $(^1/_2)$



$$\frac{x}{8-3}=\frac{y}{-12-(-24)}=\frac{1}{-3-(-4)}$$ (1/2)

or, $\frac{x}{5}=\frac{y}{12}=\frac{1}{1}$

I II III

From I and III, we get $\frac{x}{5}=\frac{1}{1}\Rightarrow x=5$ (1/2)

From II and III, we get $\frac{y}{12}=\frac{1}{1}\Rightarrow y=12$ (1/2)

Hence, required fraction is $\frac{5}{12}$. (1/2)

29) Let us suppose that $\sqrt{3}+\sqrt{5}$ is a rational number.

$\therefore$ $\sqrt{3}+\sqrt{5}$ can be written as $\frac{p}{q}$ where p and q are coprime integers.

$\Rightarrow$ $\sqrt{3}+\sqrt{5}=\frac{p}{q}$ (1)

Squaring both sides, we have $3+5+2\sqrt{3}\sqrt{5}=\frac{p^2}{q^2}$

$\Rightarrow$ $2\sqrt{15}=\frac{p^2}{q^2}-8$

$\Rightarrow$ $2\sqrt{15}=\frac{p^2-8q^2}{q^2}$

$\Rightarrow$ $\sqrt{15}=\frac{p^2-8q^2}{2q^2}$, which cannot be true. (1)

As we know that $\sqrt{15}$ is an irrational but

$\frac{p^2-8q^2}{2q^2}$ is a rational.

So, our assumption is wrong.

$\Rightarrow$ $\sqrt{3}+\sqrt{5}$ is irrational number. (1)

30) Numbers from 1 to 500, it means that the numbers1 and 500 are included and also the numbers are divisible by 2 and 5.

$\therefore$ Series is 10, 20, 30,......, 500.

$a = 10$, $d = 10$

$a_n = 500$

$\Rightarrow$ $a + (n - 1)d = 50$

$\Rightarrow$ $10 + (n - 1)10 = 500$

$\Rightarrow$ $(n - 1)10 = 490$

$\Rightarrow$ $n - 1 = 49$

$\Rightarrow$ $n = 50$ (1)

Now, $S_n = \frac{n}{2}[a+l]$ (1)

$$S_{50} = \frac{50}{2}[10+500]$$

$$S_{50} = \frac{50}{2} \times 510 = 12750$$ (1)

OR

Penalty (cost) for delay of one, two, third day are ₹ 200, ₹ 250, ₹ 300.
Now, penalty increase with next day with a difference of ₹ 50.
Required AP is ₹ 200, ₹ 250, ₹ 300, ₹ 350, ...
Here, $a = T_1 = 200$;
$d = ₹\ 50$ and $n = 30$ (1)
Amount of penalty gives after 30 days = S_{30}.

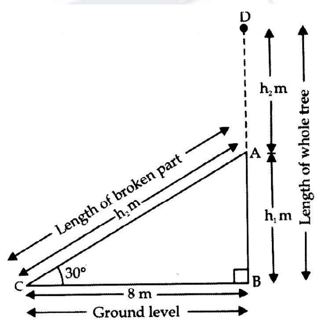
$$= \frac{n}{2}[2a+(n-1)d]$$ (1)

$$= \frac{30}{2}[2(200)+(30-1)50]$$

$= 15\ [400 + 1450]$
$= 15\ (1850) = ₹\ 27750$ (1)

Hence, ₹ 27,750 have to pay as penalty by the contractor if he has delayed the work 30 days.

31) Let BD be length of tree before storm. After storm $AD = AC$ = length of broken part of tree. The angle of elevation in this situation is 30°.



(1)

In right $\triangle ABC$,

$\frac{AB}{BC} = \tan 30°$ or $\frac{h_1}{8} = \frac{1}{\sqrt{3}}$ (1/2)

or $h_1 = \frac{8}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{8}{3}\sqrt{3}$ ----- (i) (1/2)

Also, $\frac{BC}{AC} = \cos 30°$ or $\frac{8}{h_2} = \frac{\sqrt{3}}{2}$ (1/2)

or $h_2 = \frac{8 \times 2}{\sqrt{3}} \times \frac{16}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$h_2 = \frac{16}{3}\sqrt{3}$ ----- (ii) (1/2)

Total height of tree of $h_1 + h_2$

$= \frac{8}{3}\sqrt{3} + \frac{16}{3}\sqrt{3}$ [Using (i) & (ii)] $= \left[\frac{8+16}{3}\right]\sqrt{3} = \frac{24}{3}\sqrt{3} = 8\sqrt{3}$ m (1/2)

Hence, height of the tree is $8\sqrt{3}$ m (1/2)

32) Let given points be :
$A(7, -2)$, $B(5, 1)$ and $C(3, k)$
Here $x_1 = 7, x_2 = 5, x_3 = 3$
$y_1 = -2, y_2 = 1, y_3 = k$
Three points are collinear if

$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$ (1)

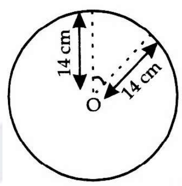
or $\quad \frac{1}{2}[7(1 - k) + 5(k + 2) + 3(-2 - 1)] = 0$

or $\quad 7 - 7k + 5k + 10 - 9 = 0$
or $\quad 2k + 8 = 0$
or $\quad -2k = -8$

or $\quad -k = \frac{-8}{-2} = 4$ (1)

Hence, $k = 4$ (1)

33) Length of minute hand of clock = Radius of circle (R) = 14 cm



(1)

We know that
$60` = 360^o$

$1` = \frac{360}{60} = 6°$ ($^1/_2$)

$5` = 6^o \times 5 = 30^o$
Angle of sector (θ) = 30^o
$\therefore$ Area swept by minute hand in 5 minutes.

$= \frac{\pi R^2 \theta}{360} = \frac{22}{7} \times 14 \times 14 \times \frac{30°}{360°}$ (1)

$= \frac{1}{12} \times 22 \times 28$

$= \frac{154}{3} \text{cm}^2 = 51.33 \text{cm}^2$ (1)

Hence, area swept by minute hand in 5 minutes is51.33 cm^2.

34) (1)

Daily wages (in ₹)	Number of workers (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$ or $u_i = \frac{x_i - 150}{20}$	$f_i u_i$
100–120	12	110	−2	−24] −38
120–140	14	130	−1	−14
140–160	8	150	0	0
160–180	6	170	1	6] 26
180–200	10	190	2	20
	$\Sigma f_i = 50$			$\Sigma f_i u_i = -12$

From given data, Assumed mean (a) = 150
and Width of the class (h) = 20

$$u = \frac{\sum f_i u_i}{\sum f_i} = \frac{-12}{50} = -0.24$$ (1)

Using formula,

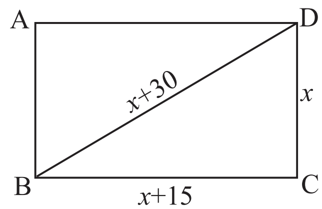
Mean $(\bar{x}) = a + h\bar{u}$ = 150 + (20) (-0.24) = 150 - 4.8 = 145.2 (1)

Hence, mean daily wages of the workers of factory is ₹ 145.20

Section - D

35) Let the shorter side of rectangular field $ABCD$ be x m.

So, diagonal $BD = (x + 30)$ m and longer side $BC = (x + 15)$ m



(1)

Now, applying Pythagoras theorem in rt. angled triangled BCD.

$$(BD)^2 = (CD)^2 + (BC)^2 \quad (1)$$

$\Rightarrow \quad (x + 30)^2 = x^2 + (x + 15)^2$

$\Rightarrow \quad x^2 + 900 + 60x = x^2 + x^2 + 225 + 30x$

$\Rightarrow \quad x^2 - 30x - 675 = 0$

$\Rightarrow \quad x^2 - 45x + 15x - 675 = 0$

$\Rightarrow \quad x(x - 45) + 15(x - 45) = 0$

$\Rightarrow \quad (x - 45)(x + 15) = 0$

$\Rightarrow \quad x = 45$ or $x = -15$ [Rejecting -15, as length can't be negative] (1)

$\therefore \quad x = 45$

Thus shorter side = 45 m

and longer side of plot = $(x + 15)$ m = (45 + 15) m = 60 m

Thus sides of rectangular field are 45 m and 60 m. (1)

36) (1)

Speed (km/h)	No. of players	cf_i
85 - 100	11	11
100 - 115	9	20
115 - 130	8	28
130 - 145	5	33
	33	

Here, N = 33

Therefore, $\frac{N}{2} = \frac{33}{2}$, which lies in 100-115. (1)

$l = 100$, $cf = 11$, $f = 9$, $h = 15$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h \quad (1)$$

$$= 100 + \left(\frac{\frac{33}{2} - 11}{9}\right) \times 15 \qquad = 100 + \frac{33 - 22}{18} \times 15$$

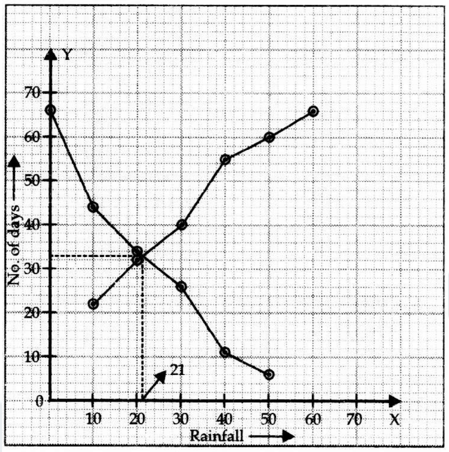
$$\text{Median } = 100 + \frac{11}{18} \times 15 = 109.67 \quad (1)$$

So, the median bowling speed = 109.67 km/h.

OR

Rainfall	No. of days	cf_1 (<)	cf_1 (>)
0 -- 10	22	22	66
10 -- 20	10	32	44
20 -- 30	8	40	34
30 -- 40	15	55	26
40 -- 50	5	60	11
50 -- 60	6	66	6
	66		

(1)



(2)

From the graph we find the median rainfall as 21 cm. (1)

37) **Given :** $\triangle ABC$, $\angle ABC < 90^o$

$AD \perp BC$.

To prove : $AC^2 = AB^2 + BC^2 - 2BC \times BD$.

Proof : ADC is right triangle at D.

$AC^2 = CD^2 + DA^2$ ---- (i) (By Pythagoras Theorem) (1)

Also, ADB is right triangle at D.

$AB^2 = AD^2 + DB^2$ ---- (ii) (By Pythagoras Theorem) (1)

From (i), we get :

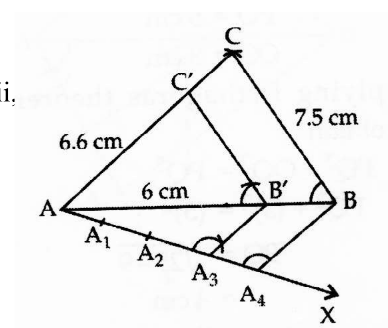
$AC^2 = AD^2 + (CB - BD)^2$

$AC^2 = AD^2 + CB^2 + BD^2 - 2CB \times BD$ (1)

or $AC^2 = (BD^2 + AD^2) + CB^2 - 2CB \times BD$

$AC^2 = AB^2 + BC^2 - 2BC \times BD$. [Using (ii)] (1)

38) **Steps of construction :-**



(a) Draw AB = 6 cm

(b) With A and B as centres takig 6.6 cm and7.5 cm as radii, draw two arcs intersecting each other at C.

(c) Join $\triangle ABC$ as the given triangle.

(d) Draw $\angle BAC$ an acute angle.

(e) Along AX draw points A_1, A_2, A_3, A_4 at equal distance such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4$.

(f) Join BA_4.

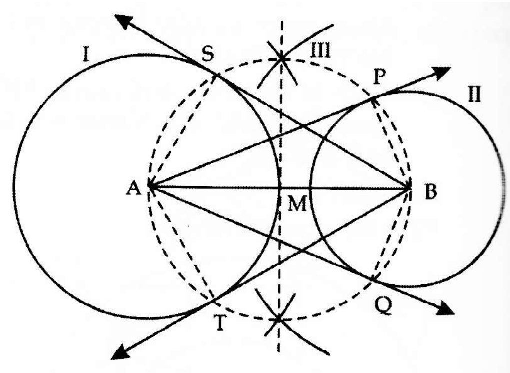
(g) Draw $A_3B` \parallel A_4B$ which intersects $AB$ at $B`$.

(h) Draw $B`C` \parallel BC$ which intersects AC at $C`$. (2 mark = for construction)

Hence, $\triangle AB`C`$ is the required triangle. (2 mark = for diagram)

OR

Steps of construction :-



($2^1/_2$ mark = for diagram)
($1^1/_2$ mark = for construction)

(a) Draw a line segment $AB = 8$ cm.
(b) With A as centre and radius 4 cm, draw a circle (I)
(c) With B as centre and radius 3 cm, draw a circle (II).
(d) Draw the perpendicular bisector of line segment AB which intersects AB at M.
(e) With M as centre and radius MA or MB, draw a circle (III) which intersects the circle (I) at S and T circle (II) at P and Q.
(f) Join AP and AQ. These are required tangents to the circle with radius 3 cm from point A.
(g) Join BS and BT. These are required tangents to the circle with radius 4 cm from point B.

39) **L.H.S.** $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right) \times \left(\frac{1}{\cos A} - \cos A\right) \qquad (^1/_2)$$

$$= \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cos A \quad \text{---- (i)} \qquad (1)$$

Now, **R.H.S.** $= \dfrac{1}{\tan A + \cot A}$

$$= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}} \qquad (^1/_2)$$

$$= \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \frac{\sin A \cos A}{1} \quad \text{---- (ii)} \qquad (1)$$

From (i) and (ii), it is clear that
∴ LHS = RHS

Hence, $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$. (1)

OR

L.H.S. $= \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$

$$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}} \qquad (^1/_2)$$

$$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}} \qquad (^1/_2)$$

$= \frac{1}{\sin\theta\cos\theta} [\because \sin^2\theta + \cos^2\theta = 1]$ $(^1/_2)$

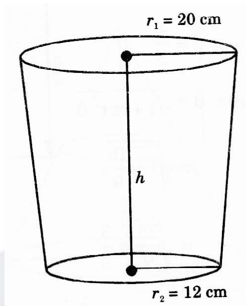
$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} [\because \sin^2\theta + \cos^2\theta = 1]$ $(^1/_2)$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$ (1)

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \tan\theta + \cot\theta$ (1)

= **R.H.S.**

40) Consider the following figure :



$(^1/_2)$

Given : Volume of the frustum is 12308.8 cm^3. Radii of the top and bottom are R = 20 cm and r = 12 cm, respectively.

Volume of the frustum is given by

$V = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$ $(^1/_2)$

12308.8 x 3 = πh ($20^2 + 12^2$ + 20 x 12)

12308.8 x 3 = πh (400 + 144 + 240)

12308.8 x 3 = πh (784)

$\frac{12308.8 \times 3}{3.14 \times 784} = h$

$\frac{3920 \times 3}{784} = h$

15 cm = h

Hence, height of the frustum is 15 cm.

Now, metal sheet required to make the frustum = Curved surface area + Area of the base of the frustum.

Curved surface area of the frustum

= $\pi(R + r)L$ $(^1/_2)$

Where, $l = \sqrt{h^2 + (R - r)^2}$ $(^1/_2)$

$l = \sqrt{15^2 + (20 - 12)^2}$

$l = \sqrt{225 + 64} = \sqrt{289} = 17$ cm $(^1/_2)$

Curved surface area of the frustum

= π (20 + 12) 17

= 544 x 3.14 = 1708.64 cm^2 $(^1/_2)$

Area of the base

= π x 12^2

= 144 x 3.14 = 452.16 cm^2 $(^1/_2)$

∴ Metal sheet required to make the frustum

= 1708.16 + 452.16 = 2160.32 cm^2 $(^1/_2)$

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