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PUNE

NAME of Student : _____

Subject : Mathematics

Chapter Test 20

Class : XII **Max. Marks :- 100**

Topic : Vector Algebra & Three Dimensional Geometry

JEE MAIN CHAPTER TEST

Marking Scheme:

(i) Each question is allotted 4 (four) marks for each correct response.

(ii) ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

Q.1 What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

(A) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (B) $-18\hat{i} - 13\hat{j} + 2\hat{k}$

(C) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (D) $6\hat{i} - 2\hat{j} + 8\hat{k}$

Q.2 With respect to a rectangular cartesian coordinate system, three vectors are expressed as : $\vec{a} = 4\hat{i} - \hat{j}, \vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -\hat{k}$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors, along the X, Y and Z-axis respectively. The unit vector $\hat{r}$ along the direction of sum of these vector is –

(A) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (B) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j} - \hat{k})$

(C) $\hat{r} = \frac{1}{3}(\hat{i} - \hat{j} + \hat{k})$ (D) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j} + \hat{k})$

Q.3 Determine the value of c so that for all real x, the vector $cx\vec{i} - 6\vec{j} - 3\vec{k}$ and $x\vec{i} + 2\vec{j} - 2cx\vec{k}$ make an obtuse angle with one another.

(A) $-4/3 < c < 0$ (B) $-1/3 < c < 0$.

(C) $-2/3 < c < 0$. (D) $-2 < c < 0$.

Q.4 Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.

(A) $\frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$ (B) $\frac{1}{3}(2\vec{i} + \vec{j} + \vec{k})$

(C) $\frac{1}{2}(\vec{i} + 2\vec{j} + \vec{k})$ (D) $\frac{1}{\sqrt{2}}(\vec{i} + \vec{j} + 2\vec{k})$

Q.5 If $\vec{a}, \vec{b}, \vec{c}$ be any three non-coplanar vectors and form a relation $(\vec{a} \times \vec{b}).\vec{c} = |\vec{a}||\vec{b}||\vec{c}|$, then the angle between $\vec{a}$ and $\vec{b}$ is-

(A) $\pi/2$ (B) 0

(B) π (D) $\pi/3$

Q.6 If $\vec{A}, \vec{B}, \vec{C}$ are non-coplanar, then the value of $(\vec{A} + \vec{B} + \vec{C}).\{(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})\}$ will be-

(A) 0 (B) $[\vec{A}\ \vec{B}\ \vec{C}]$

(C) $2[\vec{A}\ \vec{B}\ \vec{C}]$ (D) $-[\vec{A}\ \vec{B}\ \vec{C}]$

Q.7 Let a, b, c be distinct non-negative numbers. If the vectors $a\vec{i} + a\vec{j} + c\vec{k}$, $\vec{i} + \vec{k}$ and $c\vec{i} + c\vec{j} + b\vec{k}$ lie in a plane, then c is-

(A) The A.M. of a and b (B) The G..M. of a and b

(C) The H.M. of a and b (D) Equal to zero.

Q.8 The direction cosines of the line which is perpendicular to the lines whose direction cosines are proportional to (1, –1, 2) and (2, 1, –1) are –

(A) $\frac{1}{\sqrt{35}}, -\frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}$ (B) $-\frac{1}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}$

(C) $\frac{1}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}$ (D) None of these

Q.9 If $\vec{a}$, $\vec{b}$ & $\vec{c}$ are non coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ & $\vec{b}$ is :

(A) $3\pi/4$ (B) $\pi/4$

(C) $\pi/2$ (D) π

Q.10 If $\vec{a} = \vec{i} + \vec{j} + \vec{k}, \vec{b} = \vec{i} - \vec{j} + \vec{k}, \vec{c} = \vec{i} + \vec{j} - \vec{k}, \vec{d} = \vec{i} - \vec{j} - \vec{k}$ then $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is a vector which is orthogonal to both

(A) $\vec{i}$ and $\vec{j}$ (B) $\vec{j}$ and $\vec{k}$

(C) $\vec{k}$ and $\vec{i}$ (D) $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$

Q.11 If the non zero vectors $\vec{a}$ & $\vec{b}$ are perpendicular to each other then the solution of the equation, $\vec{r} \times \vec{a} = \vec{b}$ is :

(A) $\vec{r} = x\vec{a} + \frac{1}{\vec{a} \cdot \vec{a}}(\vec{a} \times \vec{b})$ (B) $\vec{r} = x\vec{b} - \frac{1}{\vec{b} \cdot \vec{b}}(\vec{a} \times \vec{b})$

(C) $\vec{r} = x(\vec{a} \times \vec{b})$ (D) none of these

Q.12 The equation of the parallel plane lying midway between the parallel planes $2x - 3y + 6z - 7 = 0$ and $2x - 3y + 6z + 7 = 0$ is –

(A) $2x - 3y + 6z + 1 = 0$ (B) $2x - 3y + 6z - 1 = 0$

(C) $2x - 3y + 6z = 0$ (D) None of these

Q.13 The equation of the plane through the point (–1, 2, 0) and parallel to the lines $\frac{x}{3} = \frac{y+1}{0} = \frac{z-2}{-1}$ and $\frac{z-2}{-1} = \frac{2y+1}{2} = \frac{z+1}{-1}$ is–

(A) $2x + 3y + 6z - 4 = 0$ (B) $x - 2y + 3z + 5 = 0$

(C) $x + y - 3z + 1 = 0$ (D) $x + 2y + 3z - 3 = 0$

Q.14 A line segment has length 63 and direction ratios are 3, –2, 6. If the line makes an obtuse angle with x–axis, the components of the line vector are

(A) 27, – 18, 54 (B) – 27, 18, 54

(C) – 27, 18, –54 (D) 27, – 18, – 54

Q.15 Equation of a plane bisecting the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$ is
(A) $5x - y - 4z - 45 = 0$ (B) $5x - y - 4z - 3 = 0$
(C) $23x + 13y + 32z - 45 = 0$ (D) $23x - 13y + 32z + 5 = 0$

Q.16 A plane passes through a fixed point (a, b, c). The locus of the foot of perpendicular to it from the origin is a sphere of radius
(A) $\sqrt{a^2 + b^2 + c^2}$ (B) $\frac{1}{2}\sqrt{a^2 + b^2 + c^2}$
(C) $a^2 + b^2 + c^2$ (D) none of these

Q.17 Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.
(A) $\sin^{-1}\left(\frac{8}{21}\right)$ (B) $\sin^{-1}\left(\frac{5}{21}\right)$
(C) $\sin^{-1}\left(\frac{7}{21}\right)$ (D) $\sin^{-1}\left(\frac{1}{21}\right)$

Q.18 The lines $\vec{r} = \vec{a} + \lambda(\vec{b} \times \vec{c})$ and $\vec{r} = \vec{b} + \mu(\vec{c} \times \vec{a})$ will intersect if –
(A) $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$ (B) $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$
(C) $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$ (D) None of these

Q.19 If P (x, y, z) is a point on the line segment joining Q (2, 3, 4) and R (3, 5, 6) such that the projections of the vector $\overrightarrow{OP}$ on the axes are $\frac{13}{5}, \frac{21}{5}, \frac{26}{5}$ respectively. The P divides QR in the ratio –
(A) 2 : 3 (B) 3 : 1
(C) 1 : 3 (D) 3 : 2

Q.20 The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the spheres and the plane –
(A) $x - y - 2z = 1$ (B) $x - 2y - z = 1$
(C) $x - y - z = 1$ (D) $2x - y - z = 1$

For Q.21-Q.25 :
The answer to each question is a NUMERICAL VALUE.

Q.21 The distance of the plane $2x - 3y + 4z - 6 = 0$ from the origin is $\frac{X}{\sqrt{29}}$. Find the value of X.

Q.22 Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ then the value of the expression $(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r}$ is equal to-

Q.23 If angle between $\vec{i} - 2\vec{j} + 3\vec{k}$ and $2\vec{i} + \vec{j} + \vec{k}$ is θ then $\sin\theta = \frac{5}{X\sqrt{7}}$. Find the value of X.

Q.24 Given $|\vec{p}| = 2, |\vec{q}| = 3$ and $\vec{p}.\vec{q} = 0$.
If $(\vec{p} \times (\vec{p} \times (\vec{p} \times (p \times \vec{q}))))$ then the vector $\vec{V} = X\vec{q}$. Find the value of X.

Q.25 The volume of the tetrahedron included between the plane $3x + 4y - 5z - 60 = 0$ and the coordinate planes is