



PARISHRAM PUBLICATIONS
PUNE

NAME of Student : _____

Subject : MATHEMATICS

Class : XI

Max. Marks :- 80

Chapter Test
1

Topic : Logarithm and functions

Instructions :

- (i) For each question in Section I, you will be awarded 3 Marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (– 1) mark will be awarded.
- (ii) For each question in Section II, you will be awarded 3 Marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (– 1) mark will be awarded.
- (iii) For each question in Section III, you will be awarded 3 Marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (– 1) mark will be awarded.
- (iv) For each question in Section IV, you will be awarded 2 marks for each row in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of 8 marks. There is no negative marking for incorrect answer(s) for this section.
- (v) For each question in Section V, you will be awarded 3 marks if you darken the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (–1) mark will be awarded.
- (vi) For each question in Section VI, you will be awarded 3 Marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, minus one (– 1) mark will be awarded.

SECTION - I

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

- Q.1** If $\{ \}$ denotes the fractional part of x , the range of the function $f(x) = \sqrt{\{x\}^2 - 2\{x\}}$ is –
 (A) ϕ (B) $[0, 1/2]$ (C) $\{0, 1/2\}$ (D) $\{0\}$
- Q.2** If $f(x)$ is a polynomial function that $f(x) \cdot f(-x) = f(2x)$, then–
 (A) No such function exists (B) $f(x)$ is linear
 (C) Number of such functions are exactly one (D) Number of such functions are exactly two
- Q.3** The least positive integral value of x , satisfying the inequality, $\log_{10}(x - 1) > \log_{10}(x - 8)$ is –
 (A) 2 (B) 6 (C) 9 (D) 10
- Q.4** The number $N = \sqrt{10^{2 + \left(\frac{1}{2}\right)\log_{16}}}$ (where base of logarithm is 10). Choose the correct option
 (1) is coprime with 9 (2) is a simple surd
 (3) is an odd composite (4) forms the sides of a right triangle with 99 and 101
 (A) 1 is correct (B) 2 & 3 are correct (C) 1 & 4 are correct (D) 1, 2 and 3 are correct
- Q.5** $f(x) = |x - 1|$, $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ and $g(x) = e^x$, $g : [-1, \infty) \rightarrow \mathbb{R}$. If the function $f \circ g(x)$ is defined, then its domain and range respectively are –
 (A) $(0, \infty)$ and $[0, \infty)$ (B) $[-1, \infty)$ and $[0, \infty)$
 (C) $[-1, \infty)$ and $\left[1 - \frac{1}{e}, \infty\right)$ (D) $[-1, \infty)$ and $\left[\frac{1}{e} - 1, \infty\right)$
- Q.6** Consider a real valued function $f(x)$ such that $\frac{1 - e^{f(x)}}{1 + e^{f(x)}} = x$. The value of a and b for which $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$ is satisfied are –
 (A) $a \in (-\infty, 1)$, $b \in \mathbb{R}$ (B) $a \in (-\infty, 1)$, $b \in (-1, \infty)$
 (C) $a \in (-1, 1)$, $b \in [-1, 1)$ (D) $a \in (-1, 1)$, $b \in (-1, 1)$
- Q.7** Which of the following function(s) have domain ?
 (A) $f(x) = \log_{x-1}(2 - [x] - [x]^2)$ (B) $g(x) = \cos^{-1}(2 - \{x\})$
 (C) $h(x) = \ln \ln(\cos x)$ (D) None of these
 (where $[.]$ and $\{.\}$ denotes greatest integer function and fractional part function respectively)

- Q.8** Period of the function $f(x) = \frac{1}{3} (\sin 3x + |\sin 3x| + [\sin 3x])$, where $[.]$ denotes greatest integer function
- (A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) π

SECTION - II

This section contains 3 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which one or more answers are correct.

- Q.9** Let $f(x) = g(x) + h(x)$ where $g(x) = \frac{1}{\pi} (\sin^{-1}x + \tan^{-1}x)$ and $h(x) = \frac{x+1}{x^2+2x+5}$ then –
- (A) Domain of $f(x)$ is $[-1, 1]$ (B) Maximum value of $g(x)$ is $3/4$
 (C) Maximum value of $h(x)$ is $1/4$ (D) Range of $f(x)$ is $\left[-\frac{3}{4}, 1\right]$

- Q.10** If function $f(x) = \log_2 \left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}} \right)$ then –

- (A) Domain of $f(x)$ is \mathbb{R} (B) Domain of $f(x)$ is $\mathbb{R} - \{n\pi\}, n \in \mathbb{I}$
 (C) Range of $f(x)$ is $[1, 2]$ (D) Range of $f(x)$ is $\mathbb{R} - [1, 2]$

- Q.11** Which of the following statements are incorrect ?

- (A) If $f(x)$ and $g(x)$ are one-one then $f(x) + g(x)$ is also one-one
 (B) If $f(x)$ and $g(x)$ are one-one then $f(x)g(x)$ is also one-one.
 (C) If $f(x)$ is odd then it is necessarily one-one.
 (D) None of these

SECTION - III

This section contains paragraph. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which only one is correct.

Passage (Q.12-Q.14)

$k(x)$ is a function such that $k(f(x)) = a + b + c + d$ where

$$a = \begin{cases} 0, & \text{if } f(x) \text{ is even} \\ -1, & \text{if } f(x) \text{ is odd} \\ 2, & \text{if } f(x) \text{ is neither even nor odd} \end{cases} \quad b = \begin{cases} 3, & \text{if } f(x) \text{ is periodic} \\ 4, & \text{if } f(x) \text{ is aperiodic} \end{cases}$$

$$c = \begin{cases} 5, & \text{if } f(x) \text{ is one-one} \\ 6, & \text{if } f(x) \text{ is many-one} \end{cases} \quad d = \begin{cases} 7, & \text{if } f(x) \text{ is onto} \\ 8, & \text{if } f(x) \text{ is into} \end{cases}$$

$A = \{x^2, e^x, \sin x, |x|\}$ all the functions in set A are defined from \mathbb{R} to \mathbb{R}

$B = \{18, 19, 16, 17\}$

$$h: \mathbb{R} \rightarrow \mathbb{R}, h(x) = \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) \text{ and } \phi: \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \rightarrow \mathbb{R}, \phi(x) = \tan x$$

- Q.12** $k(\phi(x))$ is equal to –
 (A) 15 (B) 16 (C) 17 (D) 18

- Q.13** $k(h(x))$ is equal to –
 (A) 15 (B) 16 (C) 17 (D) 18

- Q.14** If $k(x)$ is a function such that $k: A \rightarrow B, y = k(x)$, where $x \in A, y \in B$ then $k(x)$ is –
 (A) one-one onto (B) one-one into (C) many-one into (D) many-one onto

Passage (Q.15-Q.17)

Let $f(x) = f_1(x) - 2f_2(x)$, where $f_1(x) = \min\{x^2, |x|\}$, for $-1 \leq x \leq 1$
 $= \max\{x^2, |x|\}$, for $|x| > 1$

$f_2(x) = \max\{x^2, |x|\}$, for $-1 \leq x \leq 1$
 $= \min\{x^2, |x|\}$, for $|x| > 1$

and $g(x) = \begin{cases} \min\{f(t) : -3 \leq t \leq x, -3 \leq x < 0\} \\ \max\{f(t) : 0 \leq t \leq x, 0 \leq x \leq 3\} \end{cases}$

- Q.15** For $-3 \leq x \leq -1$, range of $g(x)$ is –
(A) $[-1, 3]$ (B) $[-1, -15]$ (C) $[-1, 9]$ (D) None of these
- Q.16** Number of critical points of $f(x)$ is –
(A) 1 (B) 2 (C) 3 (D) None of these
- Q.17** For $x \in (-1, 0)$, $f(x) - g(x)$ is –
(A) $x^2 - 2x + 1$ (B) $x^2 + 2x - 1$ (C) $x^2 + 2x + 1$ (D) $x^2 - 2x - 1$

SECTION - IV

This section contains match the column question . Four statements (A, B, C and D) are given in column I and four/five statements (p, q, r, s and t) in Column II. Any given statement in column I can have correct matching with one or more statement(s) given in column II.

Q.18 Match the column –

Column I

Column II

- (A) Function $f : \left[0, \frac{\pi}{3}\right] \rightarrow [0, 1]$ defined by $f(x) = \sqrt{\sin x}$ is (p) one-one function
- (B) Function $f : (1, \infty) \rightarrow (1, \infty)$ defined by $f(x) = \frac{x+3}{x-1}$ is (q) many-one function
- (C) Function $f : \left[-\frac{\pi}{2}, \frac{4\pi}{3}\right] \rightarrow [-1, 1]$ defined $f(x) = \sin x$ is (r) into function
- (D) Function $f : (2, \infty) \rightarrow [8, \infty)$ defined $f(x) = \frac{x^2}{x-2}$ is (s) onto function
(t) $f'(x) < 0$

SECTION - V

This section contains 5 questions numbered . The answer to each of the questions is a single digit integer, ranging from 0 to 9.

Q.19 If domain of $f(x) = \frac{\sin^{-1}(\sin x)}{\sqrt{-\log\left(\frac{x+4}{2}\right) \log_2\left(\frac{2x-1}{3+x}\right)}}$ is $(a, b) \cup (c, \infty)$, then find the value of $a + b + 3c$.

Q.20 If $f : (0, \infty) \rightarrow (0, \infty)$ satisfy $f(x f(y)) = x^2 y^a$ ($a \in \mathbb{R}$), then find the value of a .

Q.21 The value of $\frac{3 + \log_{10} 343}{2 + \frac{1}{2} \log_{10} \left(\frac{49}{4}\right) + \frac{1}{3} \log_{10} \left(\frac{1}{125}\right)}$ is –

Q.22 $h(x) = |kx + 5|$, domain of $f(x)$ is $[-5, 7]$, domain of $f(h(x))$ is $[-6, 1]$ and range of $h(x)$ is the same as the domain of $f(x)$, then value of k is –

Q.23 If $f(x)$ is an even function and satisfies the relation $x^2 f(x) - 2f(1/x) = g(x)$; $x \neq 0$ where $g(x)$ is an odd function. then the value of $f(2) + 2$ is –

SECTION - VI

This section contains 2 questions. Each questions contain STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

- (A) Statement- 1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement -1
- (B) Statement -1 is True, Statement -2 is True ; Statement-2 is NOT a correct explanation for Statement - 1
- (C) Statement - 1 is True, Statement- 2 is False
- (D) Statement -1 is False, Statement -2 is True

Q.24 Statement 1 : Let $f : \mathbb{R} - \{1, 2, 3\} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3}$. Then f is many-one function.

Statement 2 : If either $f'(x) > 0$ or $f'(x) < 0$, $x \in$ domain of f , then $y = f(x)$ is one-one function.

Q.25 Statement 1 : If a function $y = f(x)$ is symmetric about $y = x$, then $f(f(x)) = x$.

Statement 2 : If $f(x) = \begin{cases} x & : x \text{ is rational} \\ 1-x & : x \text{ is irrational} \end{cases}$ then $f(f(x)) = x$.