X-All, Dt.__________

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FINAL EXAMINATION
YEAR 2019-20
Std. : X (CBSE)
Subject : Mathematics
MODEL ANSWER

B

Section - A

1) (a) Let any point on the X-axis be (x, 0). Now, according to the question,

$\sqrt{(x-2)^2+(0-4)^2} = \sqrt{(x+4)^2+(0-8)^2}$ ($^1/_2$)

$\Rightarrow$ $(x-2)^2 + (-4)^2 = (x+4)^2 + (8)^2$ (Squaring both sides)

$\Rightarrow$ $x^2 + 4 - 4x + 16 = x^2 + 16 + 8x + 64$

$\Rightarrow$ $-60 = 12x$

$\therefore$ $x = -5$

Hence, the point on X-axis is (-5, 0) ($^1/_2$)

2) (c) consider, $5+\sqrt{3}+5-\sqrt{3}=10$ (rational)

and $(5+\sqrt{3})(5-\sqrt{3})=25-3=22$ (rational) (1)

3) (d) Given, BD = 8 cm and AD = 4 cm

In $\triangle ADB$ and $\triangle BDC$,

$\angle BDA = \angle CDB$ [each 90^o]

$\angle DBA = \angle DCB$ [each (90^o-A)]

$\therefore$ $\triangle ADB \sim \triangle BDC$ [By AA similarity criterion]

$\Rightarrow$ $\frac{BD}{CD}=\frac{AD}{BD}$ $\Rightarrow$ $CD=\frac{(BD)^2}{AD}$

$\Rightarrow$ $\frac{8^2}{4}=\frac{64}{4}=16\text{ cm}$ (1)

4) (b) tan 5^o tan 10^o tan 45^o tan 80^o tan 85^o

= tan 5^o tan 10^o tan 45^o cot 10^o cot 5^o [$\because$ tan (90^o - θ) = cotθ]

= (tan 5^o cot 5^o) (tan 10^o cot 10^o) tan 45^o

= 1 x 1 x 1 = 1 (1)

5) (a) Given pair of linear equation is $x + 2y = 5$ and $3x + 12y = 10$

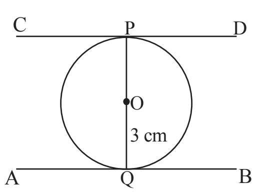
Here $a_1 = 1, b_1 = 2, c_1 = 5$

and $a_2 = 3, b_2 = 12, c_2 = 10$

Clearly $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

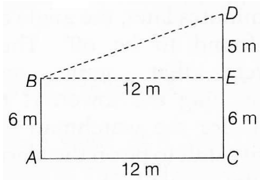
Hence, pair of linear equation has unique solution. (1)

6) (c)



Distance between two parallel tangents. $PQ = OP + OQ = 3 + 3 = 6$ cm. (1)

7) (c) Let AB and CD be the vertical poles. $AB = 6$ m, $CD = 11$ m and $AC = 12$ m



(1/2)

Draw $BE \parallel AC$, $DE = (CD - CE) = (11 - 6)$ m $= 5$ m

In right-angled $\triangle BED$ use Pythagoras theorem,

$$(BD)^2 = (BE)^2 + (DE)^2$$
$$= 12^2 + 5^2 = 169$$

$\therefore$ $BD = \sqrt{169}\text{ m} = 13\text{ m}$

Hence, distance between that tops = 13 m (1/2)

8) (a) $(3, 2)$, $\left(x, \dfrac{22}{5}\right)$ and $(8, 8)$ lie in a line and only if area of triangle = 0

$\therefore$ $\dfrac{1}{2}\left[3\left(\dfrac{22}{5} - 8\right) + x(8-2) + 8\left(2 - \dfrac{22}{5}\right)\right] = 0$

$\Rightarrow$ $3\left(\dfrac{22-40}{5}\right) + 6x + 8\left(\dfrac{10-22}{5}\right) = 0$

$\Rightarrow$ $-54 + 30x - 96 = 0$

$\Rightarrow$ $30x = 150$

$\therefore$ $x = 5$ (1)

9) (c) Given, $a_n = 3n - 8$

$a_{16} = 3 \times 16 - 8$

$\therefore$ $a_{16} = 48 - 8 = 40$ (1)

10) (a) Given, $\alpha^2 + \beta^2 = 25$ ---- (i)

and $\alpha + \beta = -7$ ---- (ii)

We know, k = Product of roots = $\alpha\beta$

From eqs. (i) and (ii) we get

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow$ $(-7)^2 = 25 + 2\alpha\beta$

$\Rightarrow$ $49 - 25 = 2\alpha\beta \Rightarrow 24 = 2\alpha\beta$

$\therefore$ $\alpha\beta = 12$ $\Rightarrow k = 12$ (1)

11) Cumulative frequency. The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class. (1)

12) (3). In cubic polynomial, the maximum number of zeroes are 3. (1)

13) $\left(\dfrac{\theta \pi r^2}{360°}\right)$. Area of sector is a circle with radius r and θ angle is given by $\dfrac{\theta \pi r^2}{360°}$. (1)

14) (0). If the graph of a quadratic polynomial does not intersect the X-axis, then the number of zeroes is zero. (1)

15) (0). In throwing of a die, the probability of getting multiple of 7 is zero. (1)

16) The given statement is false. According to the Euclid's division lemma, (1)

$a = 3q + r$

Where, $0 \leq r < 3$ and r is an integer. Therefore, the values of r can be 0, 1 or 2. (1)

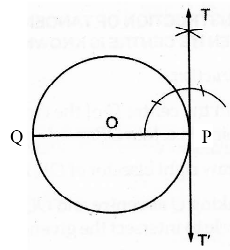
17) **Steps of Construction :-**

(a) Take a point O on the plane of the paper and draw a circle of given radius 3 cm.

(b) Take a point P on the circle and join OP.

(c) Construct $\angle OPT = 90^{\circ}$.

(d) Produce TP to T' to obtain the required tangent TPT'. (1)



18) Given : $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$

$\Rightarrow$ $\sin\alpha = \sin 30^{\circ}$

and $\cos\beta = \cos 60^{\circ}$

$\Rightarrow$ $\alpha = 30^{\circ}$ and $\beta = 60^{\circ}$ [$\because \sin 30^{\circ} = \cos 60^{\circ} = \frac{1}{2}$]

$\therefore$ $\alpha + \beta = 30^{\circ} + 60^{\circ} = 90^{\circ}$ (1)

19) Distance between the two points (0, 0) and ($a\cos\theta$, $a\sin\theta$).

$= \sqrt{(a\cos\theta - 0)^2 + (a\sin\theta - 0)^2}$ [$\because$ distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

$= \sqrt{a^2\cos^2\theta + a^2\sin^2\theta}$

$= a\sqrt{\cos^2\theta + \sin^2\theta} = a$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= a$ (1)

20) $\frac{891}{3500}$ can be written as $\frac{3^4 \times 11}{2^2 \times 5^3 \times 7}$

If denominator is of the form of 2^m x 5^n (m, n are whole number), then it is a terminating decimal. So, given fraction multiplied by minimum number 7 to make terminating decimal. (1)

Section - B

21) In $\triangle OPQ$, we have

$OQ^2 = OP^2 + PQ^2$

$(PQ + 1)^2 = OP^2 + PQ^2$ [$OQ - PQ = 1 \Rightarrow OQ = 1 + PQ$]

$PQ^2 + 2PQ + 1 = OP^2 + PQ^2$

$2PQ + 1 = 49$

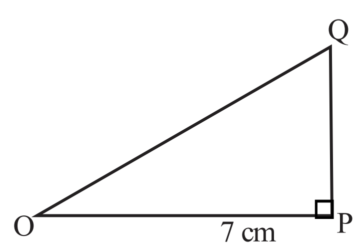
$2PQ = 48$

$PQ = 24$ cm ($^1/_2$)

$OQ - PQ = 1$ cm

$OQ = (PQ + 1)$ cm $= 25$ cm ($^1/_2$)

Now $\sin Q = \frac{OP}{OQ} = \frac{7}{25}$ ($^1/_2$)



and, $\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$ ($^1/_2$)

22) Let one zeroes of the given polynomial be α. Then, the other zeroes = $-\alpha$

Given, $f(x) = 4x^2 - 8kx - 9$

Sum of zeroes = $(-\alpha) + \alpha = 0$ (1)

$$\Rightarrow \frac{-b}{a} = 0$$

$$\Rightarrow \frac{8k}{4} = 0$$

$\therefore$ $k = 0$ (1)

23) Let the radius of the circle be r cm. Then,

Diameter = $2r$ cm and circumference = $2\pi r$ cm.

It is given that the circumference exceeds the diameter by 16.8 cm.

$\therefore$ Circumference = Diameter + 16.8

$\Rightarrow$ $2\pi r = 2r + 16.8$ ($^1/_2$)

$$\Rightarrow 2 \times \frac{22}{7} \times r = 2r + 16.8 \qquad \left[\because \pi = \frac{22}{7}\right]$$

$\Rightarrow$ $44r = 14r + 16.8 \times 7$ ($^1/_2$)

$\Rightarrow$ $44r - 14r = 117.6$

$\Rightarrow$ $30r = 117.6$

$$\Rightarrow r = \frac{117.6}{30} = 3.92$$

Hence, radius = 3.92 cm. (1)

OR

Let the radius of the protractor be r cm. Then,

Perimeter = 108 cm

$$\Rightarrow \frac{1}{2}(2\pi r) + 2r = 108 \qquad \left[\because \text{Perimeter of a semi-circle} = \frac{1}{2}(2\pi r)\right]$$ (1)

$$\Rightarrow \pi r + 2r = 108 \Rightarrow \frac{22}{7} \times r + 2r = 108 \Rightarrow 36r = 108 \times 7 \Rightarrow r = 3 \times 7 = 21$$ ($^1/_2$)

$\therefore$ Diameter of the protractor = $2r$ = (2 x 21) cm = 42 cm. ($^1/_2$)

24) Here, the class 30-35 has maximum frequency.

So, it is the modal class.

$\therefore$ $l = 30, h = 5, f_1 = 10, f_0 = 9$ and $f_2 = 3$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{10-9}{2 \times 10 - 9 - 3}\right) \times 5 = 30 + \frac{1}{8} \times 5$$

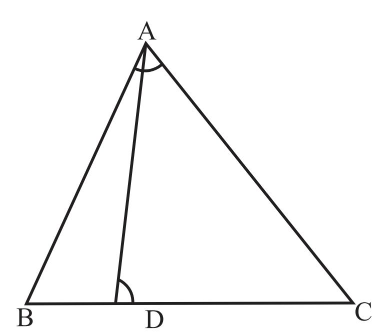
$$= 30 + 0.625 = 30.625$$

25) In $\triangle ABC$ and $\triangle DAC$, we have

$\angle ADC = \angle BAC$ and $\angle C = \angle C$

Therefore, by AA - criterion of similarity, we have

$\triangle ABC \sim \triangle DAC$ ($^1/_2$)

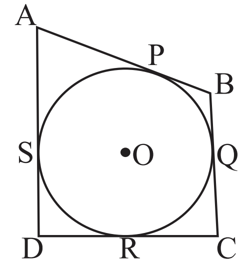


$$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$$ ($^1/_2$)

$$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD} \qquad \text{or}$$

$\Rightarrow$ $CA^2 = CB \times CD$ (1)

26) Since tangents drawn from an exterior point to a circle are equal in length.



$\therefore$ $AP = AS$ (From A) ----- (i)

$BP = BQ$ (From B) ----- (ii) $(^1/_2)$

$CR = CQ$ (From C) ----- (iii)

and, $DR = DS$ (From D) ----- (iv) $(^1/_2)$

Adding (i), (ii), (iii) and (iv) we get

$AP + BP + CR + DR = AS + BQ + CQ + DS$

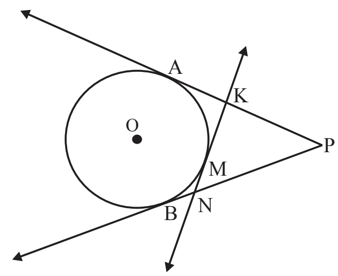
$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC$

Hence, $AB + CD = BC + DA$ (1)

OR

We know that the tangents drawn from an external point to a circle are equal in length.



$\therefore$ $PA = PB$ (From P) ----- (i)

$KA = KM$ (From K) ----- (ii)

and, $NB = NM$ (From N) ----- (iii) (1)

Adding (ii) and (iii) we get

$KA + NB = KM + NM$

$\Rightarrow$ $AK + BN = KM + MN$

$\Rightarrow$ $AK + BN = KN$ (1)

Section - C

27) Given, equation is $\dfrac{x+3}{x-2} - \dfrac{1-x}{x} = \dfrac{17}{4}$

$\Rightarrow$ $\dfrac{x(x+3)-(1-x)(x-2)}{x(x-2)} = \dfrac{17}{4}$

$\Rightarrow$ $4[x^2 + 3x - (x - 2 - x^2 + 2x)] = 17(x^2 - 2x)$ (1)

$\Rightarrow$ $4x^2 + 12x - 12x + 8 + 4x^2 = 17x^2 - 34x$

$\Rightarrow$ $8x^2 - 17x^2 + 34x + 8 = 0$

$\Rightarrow$ $9x^2 - 34x - 8 = 0$ (Dividing both sides by -1)

$\Rightarrow$ $9x^2 - 36x + 2x - 8 = 0$ (By factorisation) (1)

$\Rightarrow$ $9x(x - 4) + 2(x - 4) = 0$

$\Rightarrow$ $(x - 4)(9x + 2) = 0$

$\Rightarrow$ $x - 4 = 0$ or $9x + 2 = 0$

$x = 4$ or $x = \dfrac{-2}{9}$

Hence, the required roots of the given equation are 4 and $\dfrac{-2}{9}$. (1)

28) Let us first find the HCF of 210 and 55.

Applying Euclid's division lemma on 210 and 55, we get

$210 = 55 \text{ x } 3 + 45$ ----- (i) $(^1/_2)$

Since the remainder $45 \neq 0$. So, we now apply division lemma on the divisor 55 and the remainder 45 to get

$55 = 45 \text{ x } 1 + 10$ ----- (ii) $(^1/_2)$

We consider the divisor 45 and the remainder 10 and apply division lemma to get

$45 = 4 \text{ x } 10 + 5$ ----- (iii) $(^1/_2)$

We consider the divisor 10 and the remainder 5 and apply division lemma to get

$10 = 5 \text{ x } 2 + 0$ ----- (iv) $(^1/_2)$

We observe that the remainder at his stage is zero. So, the last divisor i.e. 5 is the HCF of 210 and 55.

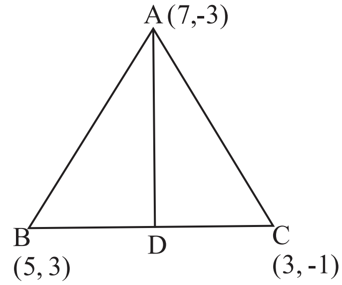
$5 = 210 \text{ x } 5 + 55y$ $(^1/_2)$

$55y = 5 - 210 \text{ x } 5 = 5 - 1050$

$55y = -1045$

$y = \dfrac{-1045}{55} = -19$ $(^1/_2)$

29) The median from a vertex of a triangle bisects the opposite side, to that vertex.
So, let AD be the median through A, then D be the mid-point of the side BC.



(1/2)

Now, coordinates of $D = \left(\frac{5+3}{2}, \frac{3-1}{2}\right) = (4,1)$ [$\because$ corodinates of mid-point of the segment (1/2)

joining (x_1, y_1) and (x_2, y_2) $= \left(\frac{x_1+x_2}{2}, \frac{y_2+y_2}{2}\right)$ (1)

and length of median AD is given by

$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ [By distance formula]

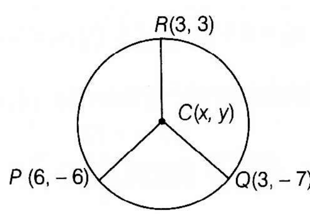
$AD = \sqrt{(4-7)^2+(1+3)^2}$

$AD = \sqrt{(-3)^2+(4)^2}$

$AD = \sqrt{9+16} = \sqrt{25} = 5$ units (1)

OR

Let $C(x, y)$ be the centre of the circle passing through the points $P(6, -6)$, $Q(3, -7)$ and $R(3, 3)$.



Then, $PC = QC = CR$ [radii of circle]

Now, $PC = QC \Rightarrow (PC)^2 = (QC)^2$ [squaring both sides]

$\Rightarrow$ $(x-6)^2 + (y+6)^2 = (x-3)^2 + (y+7)^2$ [$\because$ distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$] (1)

$\Rightarrow$ $x^2 - 12x + 36 + y^2 + 12y + 36$

$= x^2 - 6x + 9 + y^2 + 14y + 49$ [$\because$ $(a-b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow$ $-12x + 6x + 12y - 14y + 72 - 58 = 0$

$\Rightarrow$ $-6x - 2y + 14 = 0$

$\Rightarrow$ $3x + y - 7 = 0$ [dividing by -2] ---- (i)

and $QC = CR \Rightarrow (QC)^2 = (CR)^2$ [squaring both sides]

$\Rightarrow$ $(x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2$

$\Rightarrow$ $(y+7)^2 = (y-3)^2$ (1)

$\Rightarrow$ $y^2 + 14y + 49 = y^2 - 6y + 9$

$\Rightarrow$ $20y + 40 = 0$

$\Rightarrow$ $y = -\frac{40}{20} = -2$ ---- (ii)

On putting $y = -2$ in Eq. (i) we get

$3x - 2 - 7 = 0 \Rightarrow 3x = 9$

$\therefore$ $x = 3$

Hence, the centre of cirlce is (3, -2) (1)

30) We have

L.H.S. $=\dfrac{\sin\theta}{1-\cos\theta}$

$\Rightarrow \quad =\dfrac{\sin\theta}{1-\cos\theta}\times\dfrac{(1+\cos\theta)}{(1+\cos\theta)}$ [Multiplying numerator and denominator by (1+cos θ)] (1)

$\Rightarrow \quad =\dfrac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

$\Rightarrow \quad =\dfrac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$ [$\therefore$ $1 - \cos^2\theta = \sin^2\theta$] (1)

$\Rightarrow \quad =\dfrac{1+\cos\theta}{\sin\theta}$

$\Rightarrow \quad =\dfrac{1}{\sin\theta}+\dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow \quad = \operatorname{cosec}\theta + \cot\theta$

= R.H.S. (1)

31) Since one card is drawn from 52 well-shuffled cards.

$\therefore$ Total number of possible outcomes = 52

(a) Since, there are 2 queens of black colour.

$\therefore$ *P*(getting a queen of black colour)

$=\dfrac{2}{52}=\dfrac{1}{26}$ (1)

(b) In each suit, there are 2 cards with number 5 amd 6. So, that such cards are 4 times 2 = 8.

$\therefore$ *P*(getting a card with number 5 or 6)

$=\dfrac{8}{52}=\dfrac{2}{13}$ (1)

(c) In each suit, there are 6 cards with number less than 8, namely 2, 3, 4, 5, 6 and 7.

$\therefore$ *P*(getting a card with number less than 8)

$=\dfrac{4\times 6}{52}=\dfrac{24}{52}=\dfrac{6}{13}$ (1)

32) We know that the system of equations

$a_1x + b_1y = c_1 \qquad a_2x + b_2y = c_2$

has infinitely many solutions, if

$\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$ (1)

Therefore, the given system of equations will have infinitely many solutions, if

$\dfrac{2}{k+2}=\dfrac{3}{6}$ and $\dfrac{3}{6}=\dfrac{4}{3k+2}$ (1)

$\Rightarrow \quad \dfrac{2}{k+2}=\dfrac{1}{2}$ and $\dfrac{1}{2}=\dfrac{4}{3k+2}$

$\Rightarrow \quad k+2=4$ and $3k+2=8$

$\Rightarrow \quad k=2$ and $k=2$

$\Rightarrow \quad k=2$ (2)

Hence, the given system of equations will have infinitely many solutions, if $k = 2$

OR

Let the digit in the unit's place be x and the digit at the ten's place by y. Then,

Number = $10y + x$

The number obtained by reversing the order of the digits is $10x + y$.

According to the given conditions, we have

$\Rightarrow$ $(10y + x) + (10x + y) = 121$

$\Rightarrow$ $11\,(x + y) = 121$

$\Rightarrow$ $x + y = 11$

and, $x - y = \pm 3$ [$\because$ Difference of digits is 3] (1)

Thus, we have the following sets of simultaneously equations

$x + y = 11$ ---- (i) $x + y = 11$ ---- (iii)

and, $x - y = 3$ ---- (ii) $x - y = -3$ ---- (iv)

On solving equation (i) and (ii), we get $x = 7, y = 4$

On solving equation (iii) and (iv), we get $x = 4, y = 7$ (1)

When $x = 7, y = 4$, we have

Number $= 10y + x = 10 \times 4 + 7 = 47$

When $x = 4, y = 7$, we have

Number $= 10y + x = 10 \times 7 + 4 = 74$

Hence, the required number is either 47 or, 74. (1)

33) We have,

r = radius of the base of the cone = 2.1 cm

h = height of the cone = 8.4 cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (2.1)^2 \times 8.4 \text{ cm}^3$ (1)

Let R cm be the radius of the sphere obtained by recasting the melted cone. Then,

Volume of the sphere $= \frac{4}{3}\pi R^3$ (1)

Since the volume of the material in the form of cone and sphere remains the same.

$\therefore$ $\frac{4}{3}\pi R^3 = \frac{1}{3} \times \pi \times (2.1)^2 \times 8.4$ ($^1/_2$)

$\Rightarrow$ $R^3 = \frac{(2.1)^2 \times 8.4}{4} = (2.1)^3$

$\Rightarrow$ $R = 2.1$

Hence, the radius of the sphere is 2.1 cm. ($^1/_2$)

34) The value of four prizes form an A.P. with common difference $d = -20$ the sum of whose terms is 280.

Let the value of first prize be ₹ a. Then, (1)

Sum = 280

$\Rightarrow$ $\frac{4}{2}[2a + (4-1) \times -20] = 280$ (1)

$\Rightarrow$ $2\,(2a - 60) = 280$

$\Rightarrow$ $a - 30 = 70$

$\Rightarrow$ $a = 100$

Hence, the values of 4 prizes are ₹ 100, ₹ 80, ₹ 60 and ₹ 40. (1)

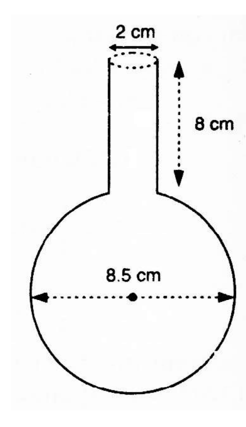
Section - D

35) We have, h = Length of the cylindrical neck = 8 cm

r = Radius of the cylindrical neck = 1 cm

Volume of the cylindrical neck

$= \pi r^2 h = \pi \times 1^2 \times 8 \text{ cm}^3 = 8\pi \text{ cm}^3$ (1)



Volume of the spherical part $= \frac{4\pi}{3} \times \left(\frac{8.5}{2}\right)^3 \text{cm}^3$

$= \frac{4\pi}{3} \times (4.25)^3 \text{cm}^3$ (1)

Amount of water in the vessel $=\left[8\pi+\frac{4\pi}{3}\times(4.25)^3\right]\text{cm}^3$ (1)

$$=\pi\left[8+\frac{4}{3}\times(4.25)^3\right]\text{cm}^3$$

$$=3.14\times\left[8+\frac{4}{3}\times 4.25\times 4.25\times 4.25\right]\text{cm}^3$$

$= 3.14 \times (8 + 102.354)\ \text{cm}^3$

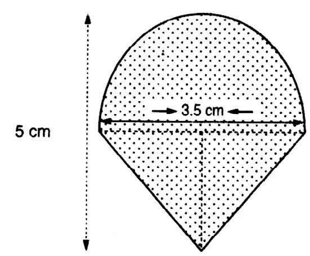
$= 346.511\ \text{cm}^3 = 346.5\ \text{cm}^3$ (1)

Hence, the volume found by the child is not correct.

OR

We have, r = Radius of hemispherical portion of the lattu $=\frac{3.5}{2}\text{cm}=\frac{7}{4}\text{cm}$ ($^1/_2$)

r = Radius of the conical portion $=\frac{3.5}{2}\text{cm}=\frac{7}{4}\text{cm}$ ($^1/_2$)



H = Height of the conical portion $=\left(5-\frac{3.5}{2}\right)\text{cm}=\frac{13}{4}\text{cm}$ ($^1/_2$)

$\therefore$ l = Slant height of the conical part $=\sqrt{r^2+h^2}$ ($^1/_2$)

$$l=\sqrt{\left(\frac{7}{4}\right)^2+\left(\frac{13}{4}\right)^2}=\sqrt{\frac{49+169}{4}}\ \text{cm}=\sqrt{\frac{218}{4}}\ \text{cm}==3.69\ \text{cm}=3.7\ \text{cm}$$ ($^1/_2$)

Total surface area of the top $= 2\pi r^2 + \pi rl = \pi r\,(2r + l)$ ($^1/_2$)

$$=\frac{22}{7}\times\frac{7}{4}\left(2\times\frac{7}{4}+3.7\right)\text{cm}^2$$ ($^1/_2$)

$$=\frac{11}{2}(3.5+3.7)\text{cm}^2=\frac{11}{2}\times 7.2\text{cm}^2$$

$= 11 \times 3.6\ \text{cm}^2 = 39.6\ \text{cm}^2$ ($^1/_2$)

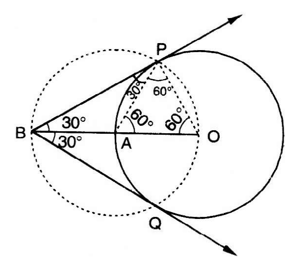
36) **Steps of construction :-** **(2 marks = construction & 2 marks = diagram)**

(a) Take a point O on the plane of the paper and draw a circle of radius OA = 5 cm.

(b) Produce OA to B such that OA = AB = 5 cm.

(c) Taking A as the centre draw a circle of radius AO = AB = 5 cm. Suppose it cuts the circle drawn in step (a) at P and Q.

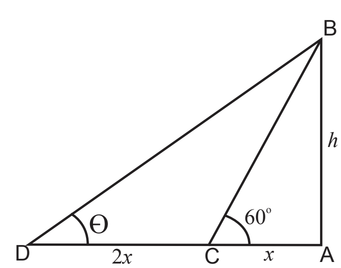
(d) Join BP and BQ to get the desired tangents.



37) Let AB be the flag-staff and let x = AC be the length of its shadow when the sun rays meet the ground at an angle of 60°. Let θ be the angle between the sun rays and the ground when the length of the shadow of the flag-staff is AD = 3x. Let h be the height of the flag-staff. (1)

In $\triangle CAB$, we have $\tan 60°=\frac{AB}{AC}$ (1)

$\Rightarrow$ $\tan 60°=\frac{h}{x}$ $\Rightarrow$ $\sqrt{3}=\frac{h}{x}\Rightarrow h=\sqrt{3}x$



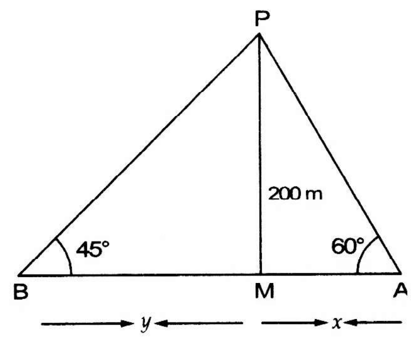
In $\triangle DAB$, we have $\tan\theta=\frac{AB}{AD}$ (1)

$\Rightarrow\tan\theta=\frac{h}{3x}\Rightarrow\tan\theta=\frac{\sqrt{3}x}{3x}\Rightarrow\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\tan\theta=\tan 30°\Rightarrow\theta=30°.$ $[\therefore h=\sqrt{3}x]$ (1)

Thus, the angle between the sun rays and the ground is 30° at the time of longer shadow.

OR

Let P be the position of the aeroplane and let A and B be two points on the two banks of a river such that the angle of depression at A and B are 60° and 45° respectively. Let $AM = x$ metres and $BM = y$ metres. We have to find AB. (1)



In $\triangle AMP$, we have $\tan 60^\circ = \dfrac{PM}{AM}$

$$\Rightarrow \quad \sqrt{3} = \frac{200}{x}$$

$$\Rightarrow \quad 200 = \sqrt{3}x \Rightarrow x = \frac{200}{\sqrt{3}} \qquad \text{---- (i)}$$ (1)

In $\triangle BMP$, we have $\tan 45^\circ = \dfrac{PM}{BM}$

$$\Rightarrow \quad 1 = \frac{200}{y} \Rightarrow y = 200 \qquad \text{---- (ii)}$$ (1)

From equation (i) and (ii) we get

$$AB = x + y = \frac{200}{\sqrt{3}} + 200 = 200\left(\frac{1}{\sqrt{3}} + 1\right) = 315.4 \text{ metres}$$ (1)

Hence, the width of the river is 315.4 metres.

38) Suppose that one man alone can finish the work in x days and one boy alone can finish it in y days. Then,

One man's one day's work $= \dfrac{1}{x}$ One boy's one day's work $= \dfrac{1}{y}$ ($^1/_2$)

Eight men's one day's work $= \dfrac{8}{x}$ 12 boy's one day's work $= \dfrac{12}{y}$ ($^1/_2$)

Since 8 men and 12 boys can finish the work in 10 days

$$= 10\left(\frac{8}{x} + \frac{12}{y}\right) = 1 \Rightarrow \frac{80}{x} + \frac{120}{y} = 1 \qquad \text{----- (i)}$$ ($^1/_2$)

Again, 6 men and 8 boys can finish the work in 14 days.

$$= 14\left(\frac{6}{x} + \frac{8}{y}\right) = 1 \Rightarrow \frac{84}{x} + \frac{112}{y} = 1 \qquad \text{---- (ii)}$$ ($^1/_2$)

Putting $\dfrac{1}{x} = u$ and $\dfrac{1}{y} = v$ in equations (i) and (ii), we get

$80u + 120v - 1 = 0$
$84u + 112v - 1 = 0$

By using cross multiplication, we have

$$\frac{u}{-120+112} = \frac{-v}{-80+84} = \frac{1}{80\times 112 - 120\times 84}$$ ($^1/_2$)

$$\frac{u}{-8} = \frac{v}{-4} = \frac{1}{-1120}$$

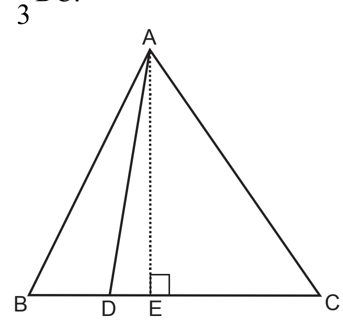
$$u = \frac{-8}{-1120} = \frac{1}{140} \text{ and } v = \frac{-4}{-1120} = \frac{1}{280}$$ ($^1/_2$)

Now, $u = \dfrac{1}{140} \Rightarrow \dfrac{1}{x} = \dfrac{1}{140} \Rightarrow x = 140$ ($^1/_2$)

and, $v = \dfrac{1}{280} \Rightarrow \dfrac{1}{y} = \dfrac{1}{280} \Rightarrow y = 280$ ($^1/_2$)

Thus, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

39) Let ABC be an equilateral triangle and let D be a point on BC such that $BD = \frac{1}{3}BC$. $(^1/_2)$



Draw $AE \perp BC$. Join AD.

In $\triangle AEB$ and $\triangle AEC$, we have $AB = AC$,

$\angle AEB = \angle AEC = 90^\circ$ $(^1/_2)$

and, $AE = AE$

So, by RHS - criterion of similarity, we have

$\triangle AEB \sim \triangle AEC$ $(^1/_2)$

$\Rightarrow$ $BE = EC$

Thus, we have $BD = \frac{1}{3}BC, DC = \frac{2}{3}BC$ and $BE = EC = \frac{1}{2}BC$ ---- (i) $(^1/_2)$

Since, $\angle C = 60^\circ$. Therefore $\triangle ADC$ is an acute triangle.

$AD^2 = AE^2 + DE^2$

In ΔAEC,

$AC^2 = AE^2 + EC^2$

$\therefore AE^2 = AC^2 + EC^2$

$AD^2 = AC^2 + EC^2 + [DC - EC]^2$

$= AC^2 + DC^2$

$- 2DC \times ED$

$= AC^2 + DC^2 - 2DC \times EC$

$AD^2 = AC^2 + DC^2 - 2DC \times EC.$ $(^1/_2)$

$$AD^2 = AC^2 + \left(\frac{2}{3}BC\right)^2 - 2 \times \frac{2}{3}BC \times \frac{1}{2}BC \qquad \text{[Using (i)]} \qquad (^1/_2)$$

$$AD^2 = AC^2 + \frac{4}{9}BC^2 - \frac{2}{3}BC^2$$

$$AD^2 = AB^2 + \frac{4}{9}AB^2 - \frac{2}{3}AB^2 \qquad [\because AB = BC = AC] \qquad (^1/_2)$$

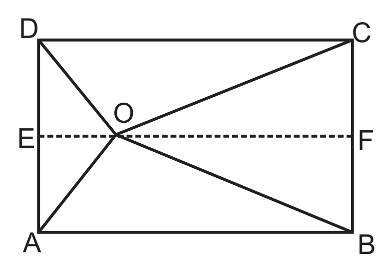
$$AD^2 = \frac{9AB^2 + 4AB^2 - 6AB^2}{9} = \frac{7}{9}AB^2$$

$9AD^2 = 7AB^2$ $(^1/_2)$

OR

Let $ABCD$ be the given rectangle and let O be a point within it. Join OA, OB, OC and OD.

Through O, draw $EOF \parallel AB$. Then $ABFE$ is a rectangle.



In right triangles $\triangle OEA$ and $\triangle OFC$, we have

$OA^2 = OE^2 + AE^2$ and $OC^2 = OF^2 + CF^2$ (1)

$OA^2 + OC^2 = (OE^2 + AE^2) + (OF^2 + CF^2)$

$OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$ $(^1/_2)$

Now, in right triangles $\triangle OFB$ and $\triangle ODE$, we have

$OB^2 = OF^2 + FB^2$ and $OD^2 = OE^2 + DE^2$ (1)

$OB^2 + OD^2 = (OF^2 + FB^2) + (OE^2 + DE^2)$

$OB^2 + OD^2 = OE^2 + OF^2 + DE^2 + BF^2$

$OB^2 + OD^2 = OE^2 + OF^2 + CF^2 + AE^2$ $[\because DE = CF$ and $AE = BF]$ $(^1/_2)$

From (i) and (ii), we get

$OA^2 + OC^2 = OB^2 + OD^2$ (1)

40) Given distribution is cumulative frequency distribution of less than type.

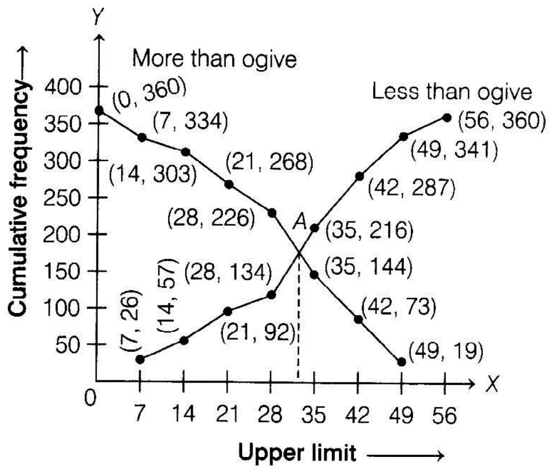
Now, we mark the upper limits along X-axis and cumulative frequencies along Y-axis, on the graph paper. Then, plot the points (7, 26), (14, 57), (21, 92), (28, 134), (35, 216), (42, 287), (49, 341) and (56, 360). Join all these points by a freehand smooth curve to obtain an ogive of less than type.

Now, let us form the cumulative frequency distribution of more than type, as shown below

Height	Frequency	Height (More than or equal to)	Cumulative Frequency
0 -- 7	26	0	360
7 -- 14	57 - 26 = 31	7	360 - 26 = 334
14 -- 21	92 - 57 = 35	14	334 - 31 = 303
21 -- 28	134 - 92 = 42	21	303 - 35 = 268
28 -- 35	216 - 134 = 82	28	268 - 42 = 226
35 -- 42	287 - 216 = 71	35	226 - 82 = 144
42 -- 49	341 - 287 = 54	42	144 - 71 = 73
49 -- 56	360 - 341 = 19	49	73 - 54 = 19

(1)

Now, we plot the points (0, 360), (7, 334), (14, 303), (21, 268), (28, 226), (35,144), (42,73) and (49, 19) on the same graph paper by choosing a suitable scale. Join all these points by a freehand smooth curve to obtain an Ogive of more than type. (1)



(1)

The two ogives intersect at point A. Now, we draw a perpendicular line from A to the X-axis, the intersection point on X-axis is 31.9. Thus, the required median is 31.9. (1)

~ 0 ~ 0 ~ 0 ~ 0 ~ 0 ~